RIOD SIMULATION RESULTS
DARK MATTER HALO COLLAPSE SCENARIOS:

Part 5: Comparing Plummer and EX10

force softening methods

Study Began May 26, 2019

https://riodsim.weebly.com/

PART 5 STUDY RESULTS:
COMPARING EX10 AND PLUMMER FORCE SOFTENING

- The study results will be presented first, see following slides for motivations, details, data and analysis of the study.
- In general, the EX10 density profiles are slightly larger within the halo interior as compared to the Plummer profiles.
- In many cases, only modest changes are seen between the density profiles at T=13.8 Gyr.
- Evolution of the system virial radius reveals some differences in the method; mostly in the virial state arrival time and some oscillatory phase differences.
- EX10 being more localized, it tracks fewer close particle pairs, resulting in reduced compute time to simulation completion. In two directly comparable scenarios run on the same computer, EX10 force softening was 1.15 and 1.19 times faster on average to complete the simulation. These two examples have the largest ratios of force softened pairs seen in this study of force softened pairs at the simulation end, 2800 and 3000, respectively.
- EX10 softening has no detrimental effects and many positive attributes and as such, is the best option for force softening in the Riod simulation.

DARK MATTER HALO COLLAPSE STUDY MOTIVATIONS

- The results presented here are part of a broad, continuing study, to examine gravitational dark-matter-like collapse scenarios. The idea was to systematically study many scenarios, while attempting to keep only modest changes between simulation runs.
- There are many motivations for beginning this study and I will address some of those below:
  - These collapse scenarios are something that my simulation was easily adapted to do and provides an interesting pastime to contemplate physics topics.
  - It was within the last decade that I discovered that the simulation could adapted and used for "real" cosmological purposes.
  - Curiosity about dark matter as a particle and how might these suspected small objects evolve from the early universe to create galactic halos. Many galactic properties arise from the need for dark matter, constant stellar circular velocities for example.
  - A healthy skepticism about dark matter as a WIMP, a small particle who is its own antiparticle, intrigued me has led to some interesting test scenarios that perhaps will be reported in future studies.
  - Curiousity if I could reproduce halo shapes seen in other, much larger studies. Dark matter halos created in simulations are often compared to density profiles like the NFW, Einasto, and Jaffe profiles.
  - Finally, these initial configurations were kept intentionally simple to provide a learning baseline.

DARK MATTER COLLAPSE STUDY:
PART 5: COMPARING PLUMMER AND EX10 FORCES SOFTENING METHODS

- The results reported here begins with the premise: How do particle collapse change when using different force softening methods. Here are presented previous results but now with the direct comparison between two force softening methods, the Plummer and what I call the EX10 derived force. More on the EX10 function later.
- As noted in previous results, from a cosmological perspective, the results presented here are scenarios that are not particularly relevant. Cosmic Microwave Background data shows that the early universe is a soup of Gaussian density fluctuations. This simulation scenarios discussed herein can best be thought of as an isolated over-density, in comoving coordinates. In addition, the initial virial density for this study is less than 200 times the current critical density, which implies (I think) that the initial densities are a bit too low, cosmologically speaking.
- The paper from Diemand et.al. (2006) inspired the particle mass and force softening length for this study.
  - Keep number of particles, what I call standard objects (SO) the same (20,000). SO mass is 4.2×10^{34} kg or 21,000 solar masses.
  - Keep the initial sphere radius distance the same at 19.0 kpc.
  - Iterative time slice is 4.33×10^{11} seconds and is the same for all simulations.

SIMULATION INITIAL KINECT ENERGY CONDITIONS

- In this study, different initial kinetic energy conditions will be referred to as the total energy ratio, $K/|E|$. Here K is the total kinetic energy and $|E|$ is the magnitude of the total energy. Note too that for bound systems the total energy in a negative quantity.
- We know that for systems of particles in virial equilibrium, the energy ratio, $ER=K/|E|=1$.
- Comparisons of data using differencing force softening methods chosen for this study, ER=0.0, 0.05, 0.10, and 0.20. These simulations will be referred to as ER000, ER005, ER010, and ER020, respectively.
- From a consideration of initial cosmological conditions, the early universe would have relative kinetic energies close to zero in a comoving group of this size.

WHAT IS FORCE SOFTENING AND WHAT METHODS ARE USED?

- In order to simulate gravitational systems, some form of force softening is required to mitigate the Newtonian force's $1/r^2$ behavior. The force will become computationally unstable once particle separation distances become small.
- In creating the Riod simulation, this was one thing that was "solved" early on by modifying the force of gravity to avoid the unpleasantness at small particle separation distances.
- Two questions arose to address this issue:
  - What separation distances are considered small?
  - How can the Newtonian force be modified?

FORCE SOFTENING: WHAT IS THE SOFTENING SCALE?

- To understand how the softening scale was found, we first need to understand the simulation time scale. Since the Riod simulation is an iterative solution to the many-body problem, a question was asked.
- What is the orbital period for two identical particles orbiting at their radii (s)? For classical gravitating particles, there is a known solution for this question. The period of orbit (τ) (curtesy of Kepler and Newton) is given by

$$\tau^2 = \frac{2\pi^2 s^3}{Gm}$$

- The period of the orbit is then divided into "N" time slices and the simulation iterative time is, $\Delta t = \tau / N$. Once this is done, for specific iterative time, the size of the particle is determined or vice versa.
- By this solution, it is understood that for distances less than twice the radial size, the simulation will produce results that are suspect and as such, forces needed to be modified to account for those situations, i.e. force softening.

FORCE SOFTENING METHODS: EX10

- The EXn family of profiles is a simple construct to model density functions using the exponential function with known (workable within the constraints of the simulation) analytic solutions for the M(r) to ρ(r) transformation. Here specifically, n=5, 10, 15, 20, 30, 40, ... Note that this density is just a (less general) variant of the Einasto profile.

$$\text{EXn}=\rho(x) = \rho_0 e^{-x^{30/n}}$$

Here, $x=r/r_s$ is a distance scaled by r_s, and ρ_0 is the density at r=0.

- The Riod simulation has been using the EX10 force softening method since 4/2014. At the time it was put into the code, I did not know of the Plummer method but a Plummer option was added as a feature in February, 2019.
- Given the EXn definition above, EX10 is the following density profile:

$$\text{EX10}=\rho(x) = \rho_0 e^{-x^{3}}$$

It can be shown that the EX10 form has solution to the Poisson equation, albeit somewhat clumsy as an analytic solution.

- Finally, when referring to EX10 force softening, the simulation uses the M(R) construct to modify interior forces ($F_<$) out to r_s (as defined above).

$$F_< = -G\,\frac{mM}{r^2} = -G\,\frac{mM(r)}{r^2}\ ;where\text{: M(x)}= 1 - e^{-x^3}$$

J. Einasto (1965), Kinematics and dynamics of stellar systems, *Trudy Inst. Astrofiz. Alma-Ata* 5, 87

PLUMMER FORCE SOFTENING

- The Plummer method of force softening has been around since 1911 and the Plummer density is given by:

$$\rho(x) = \rho_0 \frac{1}{(1+x^2)^{5/2}}$$

Again, $x=r/r_s$ is a distance scaled by r_s, and ρ_0 is the density at r=0. It can also be shown that the Plummer density has a Poisson equation solution, but this solution is a bit more elegant than for the EX10 function.

- The M(r) solution is given by:

$$F_< = -G\frac{mM}{r^2} = -G\frac{mM(r)}{r^2}\ ; where: \mathbf{M(x)} = \frac{x^2}{(1+x^2)^{3/2}}$$

- Let's compare Plummer and EX10. As the both converge to the Newtonian value at $r >> r_s$. Let's use the comparison point where $F_</F_N=0.999$. For EX10, this happens at $x=r/r_s=1.9$ and for Plummer, $x=r/r_s=39$.

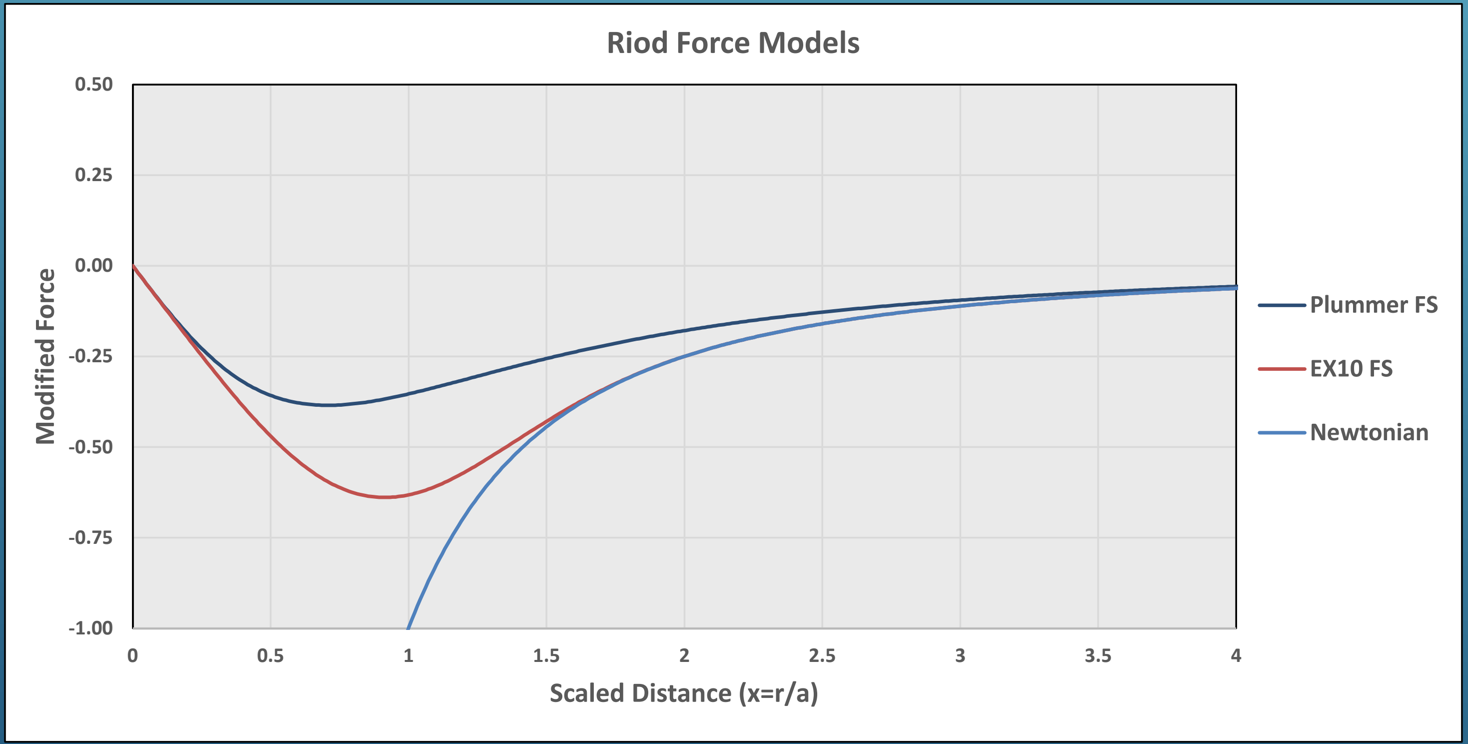
Plummer, H. C. (1911), On the problem of distribution in globular star clusters, *Mon. Not. R. Astron. Soc.* **71**, 460

PROPERTIES OF PLUMMER AND EX10 FORCE MODIFICATIONS

- Some of the properties of these force modifications are:

1. Both Plummer and EX10 interior forces go to zero as r approaches zero with the same slope with x<0.2.
2. Both force modifications will default to the Newtonian force for large distances.
3. The true difference between these methods becomes evident at where their modifications differ. The EX10 method is 20 times more localized than Plummer.

- The plot to the right clearly shows the EX10 function converging to the Newtonian force at x>2 but the Plummer force doesn't converge back to Newtonian force until x>39.
- The slope of the force is gentler in Plummer than EX10 but sufficient testing has shown that there is no ill behaviors using EX10.
- One large advantage for using the EX10 method is, the simulation code has to track fewer softening pairs and thus computationally more efficient.



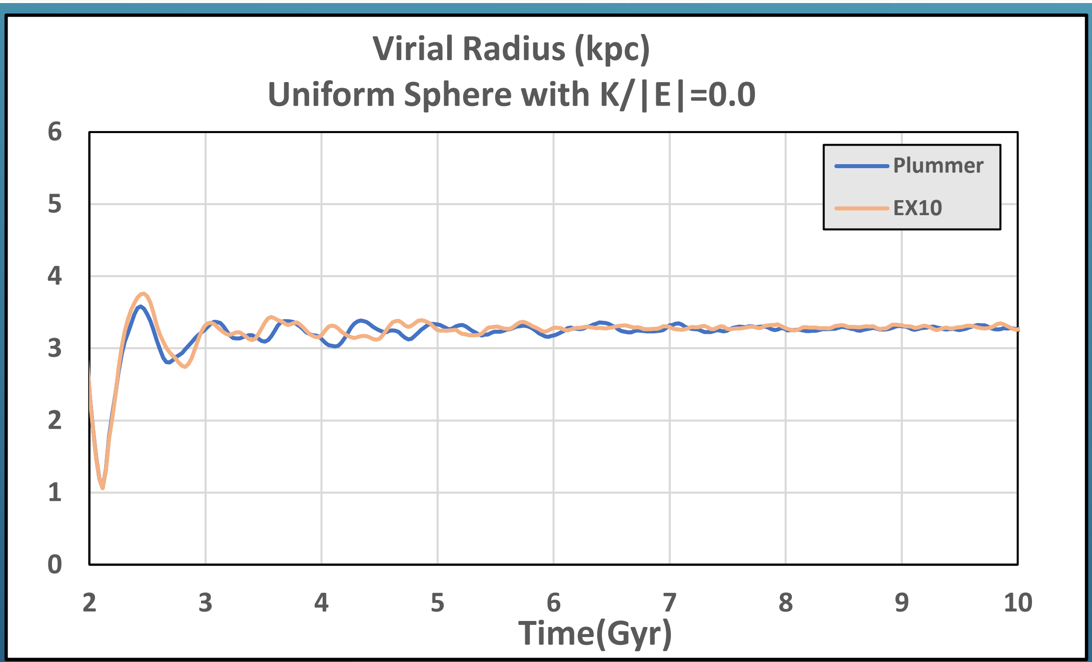
FORCE SOFTENING COMPARISON RESULTS

- Results will be presented for the following collapse scenarios. The mass shell referred to below is defined as a uniform distribution between two radii, where the inner radius is a percentage of the outer.
  - Uniform Sphere for $K/|E|=0.0, 0.05, 0.20$
  - 60% Mass Shell for $K/|E|=0.0, 0.05, 0.20$
  - 80% Mass Shell for $K/|E|=0.0, 0.10, 0.20$
- In each specific initial distribution
  - Simulations with different energy ratios begin with all particles in the same initial positions.
  - All initial virial radii are the same.

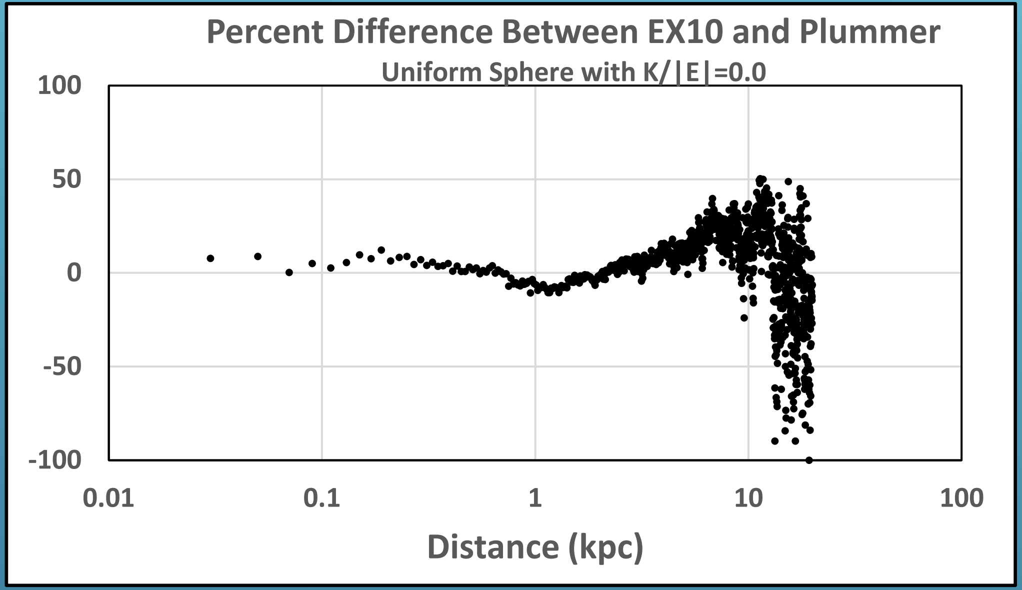
STUDY RESULTS:
COLLAPSING SPHERE WITH K/|E|=0.0

- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 10 Gyr, well after each system has achieved a virial condition.
- Note that the virial radius tracks between the Plummer and EX10 simulations nearly perfectly until about 2.3 Gyr. They stay in phase for an additional Gyr before following different tracks.
- The EX10 virial radius initially has more extreme changes between 2 and 3 Gyr but then seems to reach a virial state more quickly than the Plummer case.
- Note that the final virial radius for each case was expected to be 7.9 kpc. The difference in the expected and actual final virial radius is due to the high percentage of ejected particles from the system.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	15,460	15,694
Ave. number of force-softened pairs	7.21×10^7	3.18×10^5
Ave. number of particles lost	4540	4306
Final average virial radius	3.28 kpc	3.30 kpc



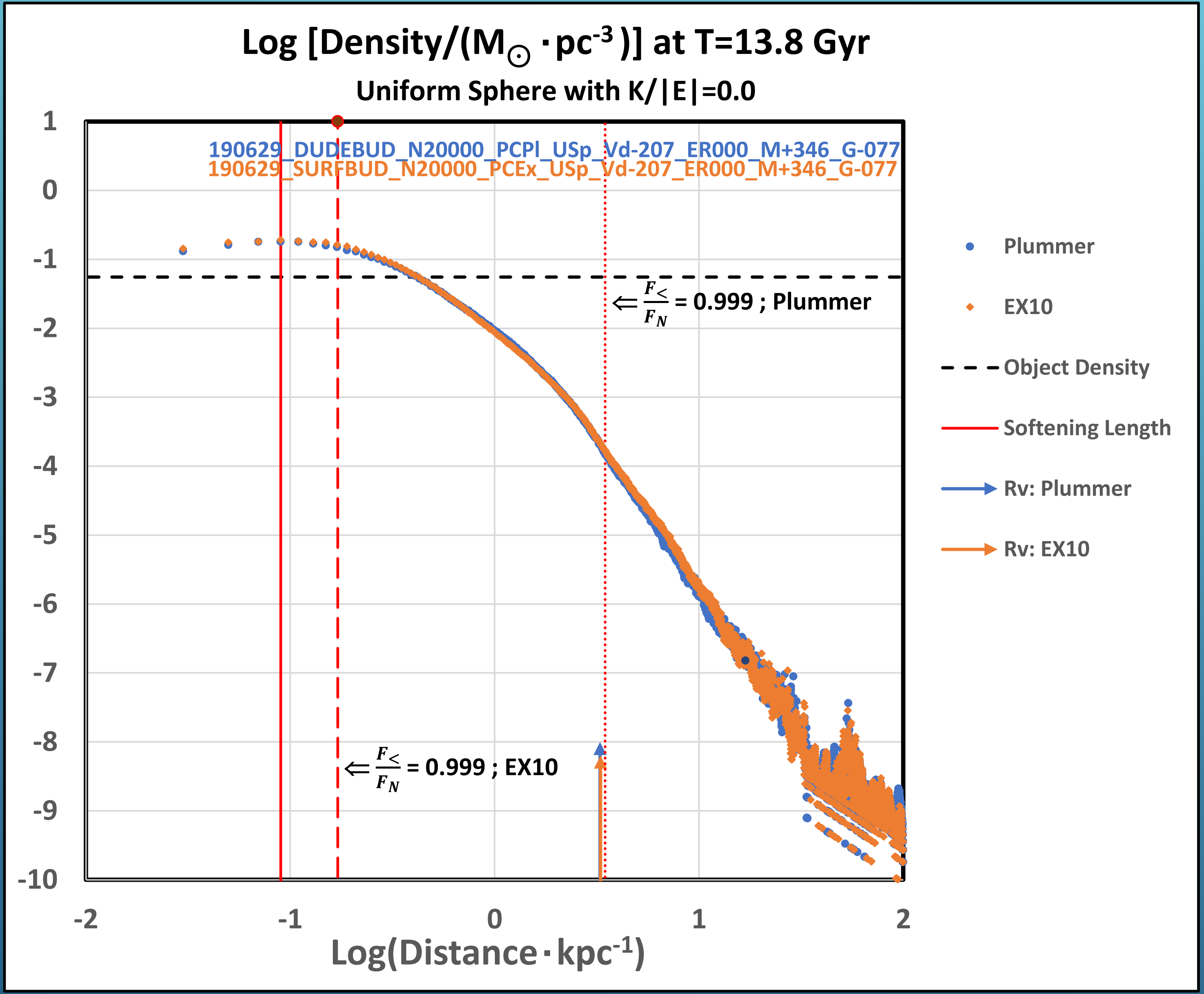
STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING SPHERE WITH K/|E|=0.0



- The density profiles for the two force softening method are remarkably similar, from extreme interior, out to over 10 kpc.
- The plot above highlights the differences in the density profiles. The EX10 simulation density is consistently larger throughout the range except around the 0.7 to 2.5 kpc region.

Notes:

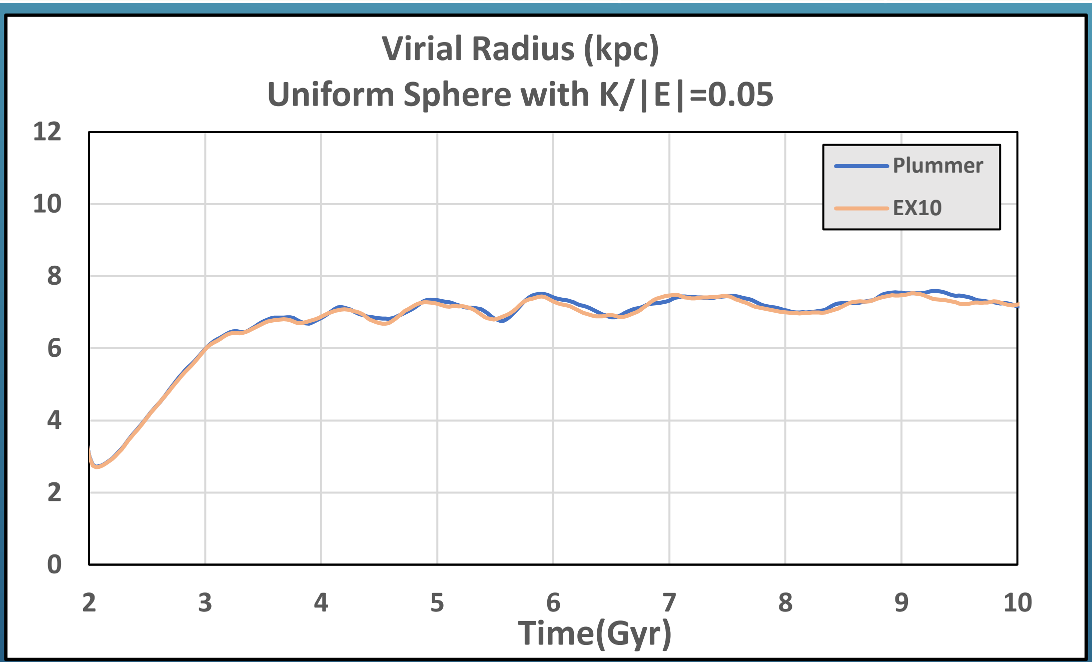
- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are over 200 times the number of force softened pairs for the Plummer vs. EX10 method at 13.7 Gyr!



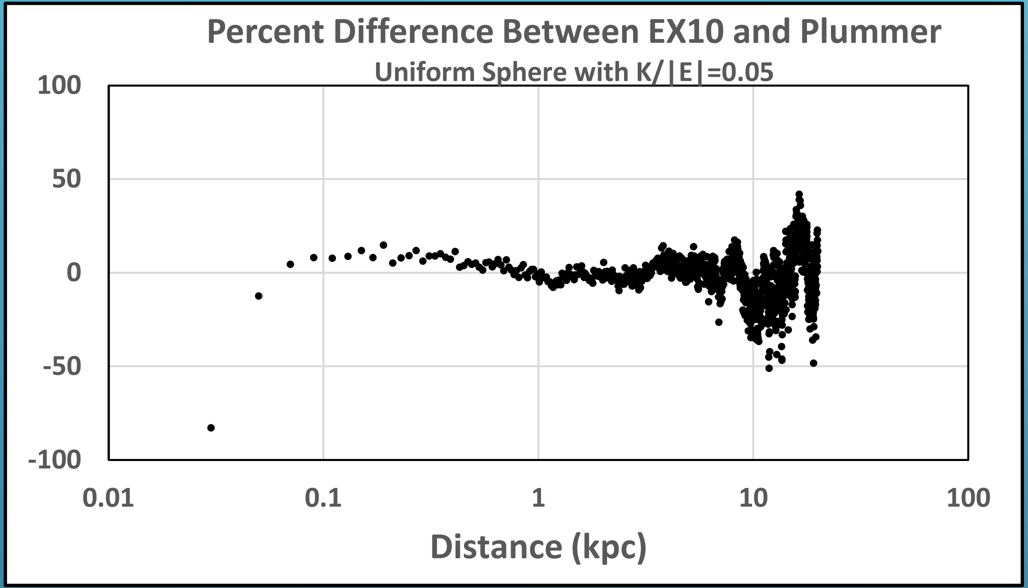
STUDY RESULTS:
COLLAPSING SPHERE WITH K/|E|=0.05

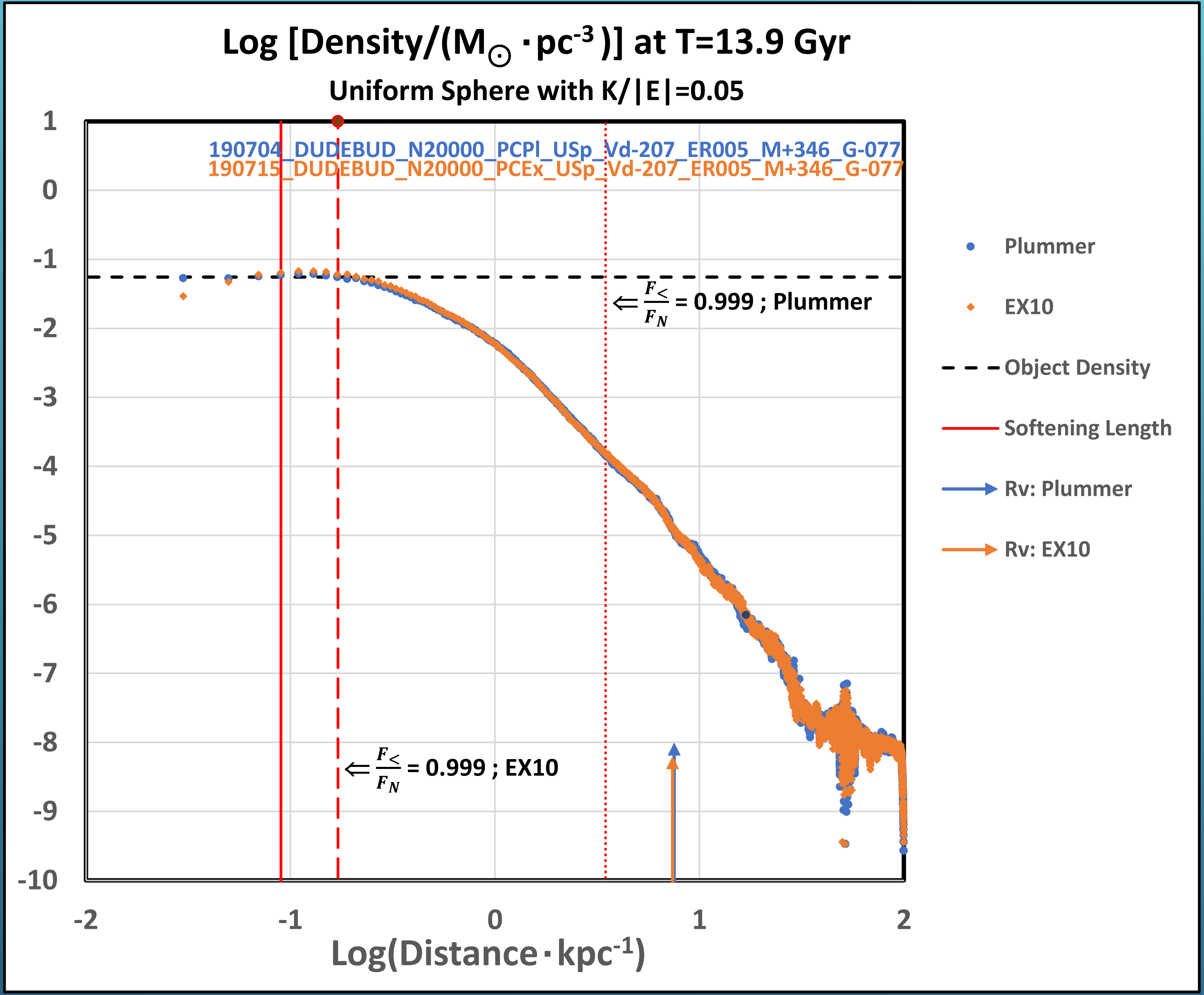
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 10 Gyr, well after each system has achieved a virial condition.
- Note that the virial radius tracks between the Plummer and EX10 simulations nearly perfectly until about 3 Gyr. The larger oscillations stay in phase for an extended period and don't significantly deviate until 9 Gyr.
- Note, the final virial radius for each case was expected to be 7.9 kpc. The difference in the expected and actual final virial radius is due to the high percentage of ejected particles from the system. However, it is interesting that there are similar numbers of particles lost as the zero kinetic energy example but the virial radius is much closer to the expected value.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	15,526	15,480
Ave. number of force-softened pairs	2.67×10^{7}	6.98×10^{4}
Ave. number of particles lost	4474	4521
Final average virial radius	7.52 kpc	7.40 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING SPHERE WITH K/|E|=0.05





- The density profiles for the two force softening method are remarkably similar, from extreme interior, even well beyand 10 kpc.
- The EX10 simulation density is consistently slightly larger out to 9 kpc .

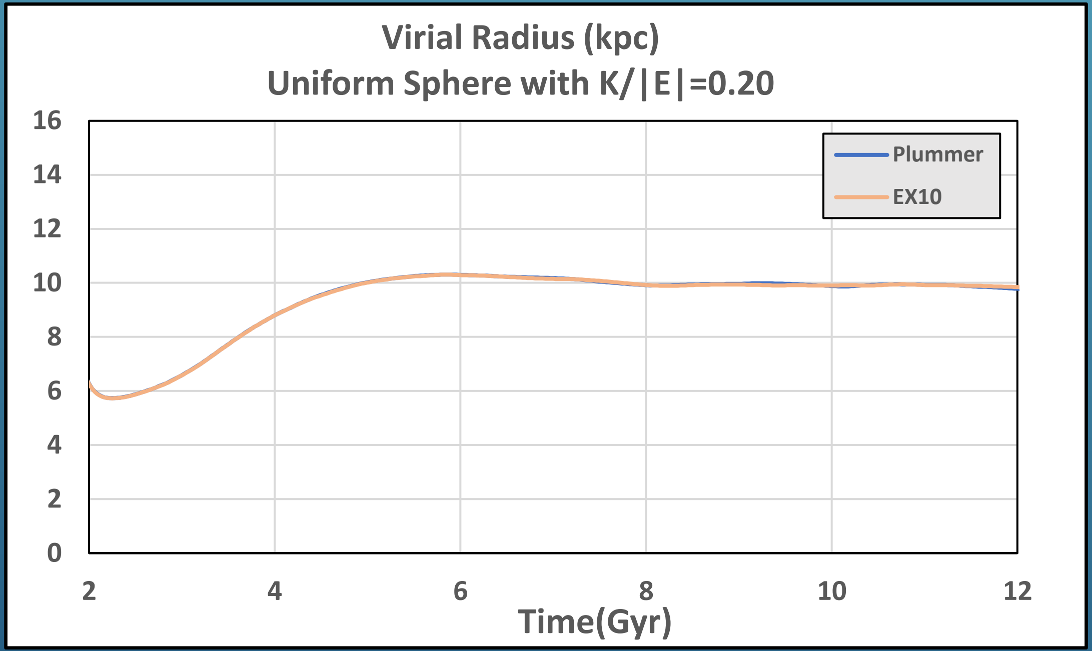
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are nearly 400 times the number of force softened pairs, Plummer over EX10.

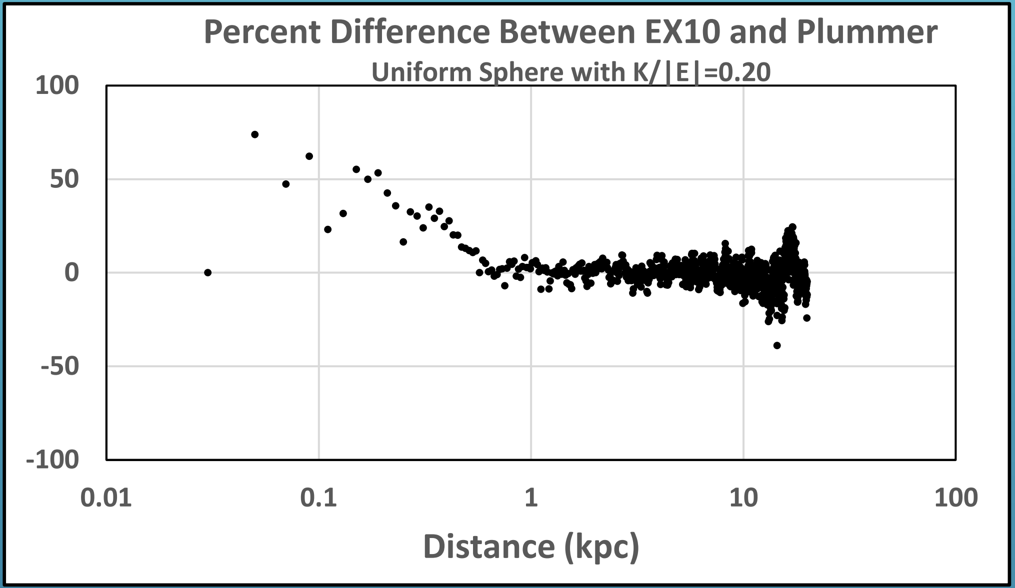
STUDY RESULTS:
COLLAPSING SPHERE WITH K/|E|=0.20

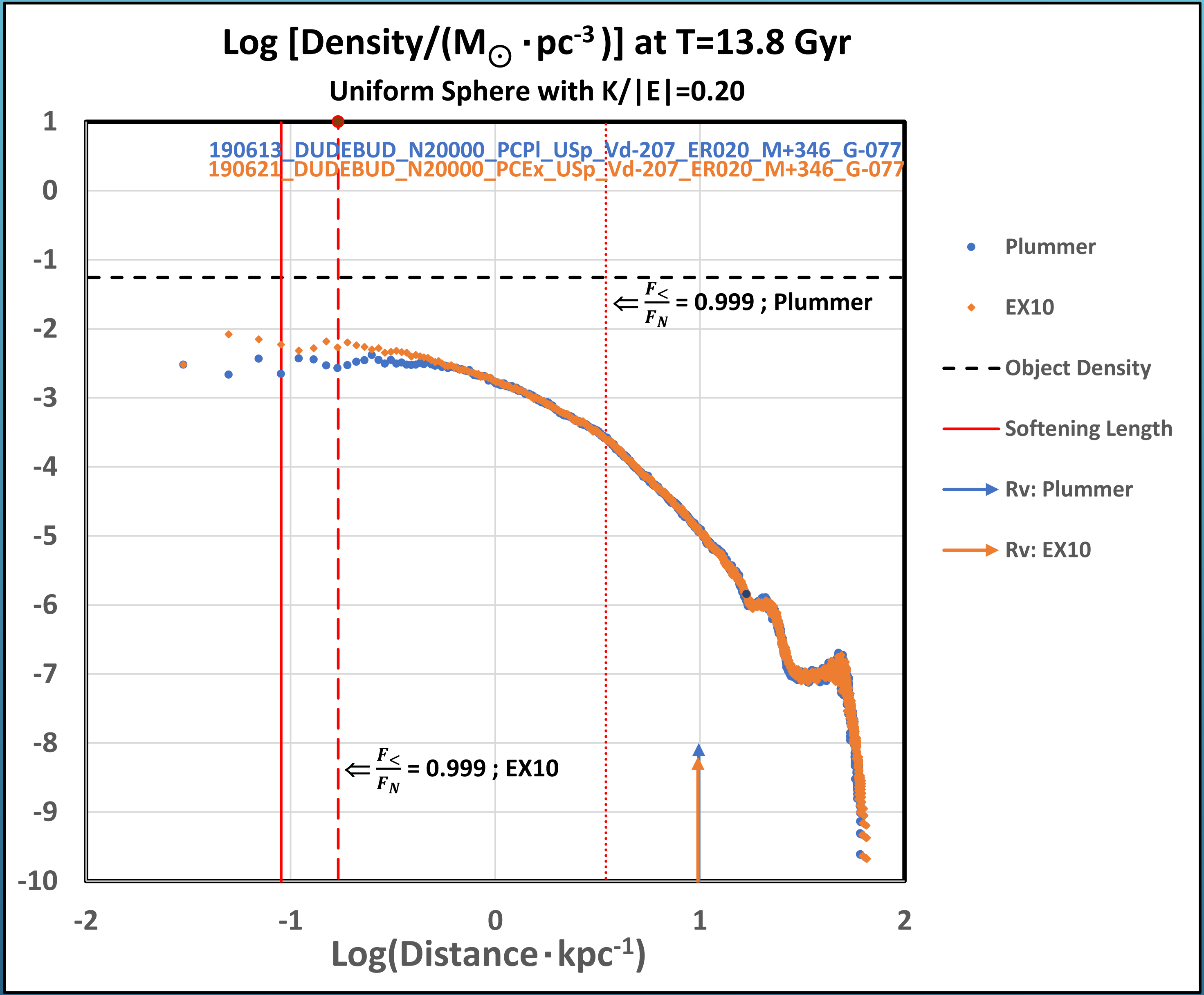
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 12 Gyr, well after each system has achieved a virial condition.
- Note that the virial radius tracks between the Plummer and EX10 simulations nearly perfectly throughout the time displayed.
- Note that the final virial radius for each case was expected to be 9.46 kpc. In this case, difference in the expected and actual final virial radius is probably because these systems are still in virial fluctuation. In addition, the histograms were averaged over 0.5 Gyr, the temporal variations of the virial radius are larger than the averaging interval.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	20,000	20,000
Ave. number of force-softened pairs	1.09×10^7	5.41×10^3
Ave. number of particles lost	0	0
Final average virial radius	9.86 kpc	9.78 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING SPHERE WITH K/|E|=0.20





- The density profiles for the two force softening methods are nearly identical beyond 0.7 kpc. However, the interior region shows the only significant differences in these methods.
- The Plummer simulation flattens in the interior faster than the EX10 simulation.

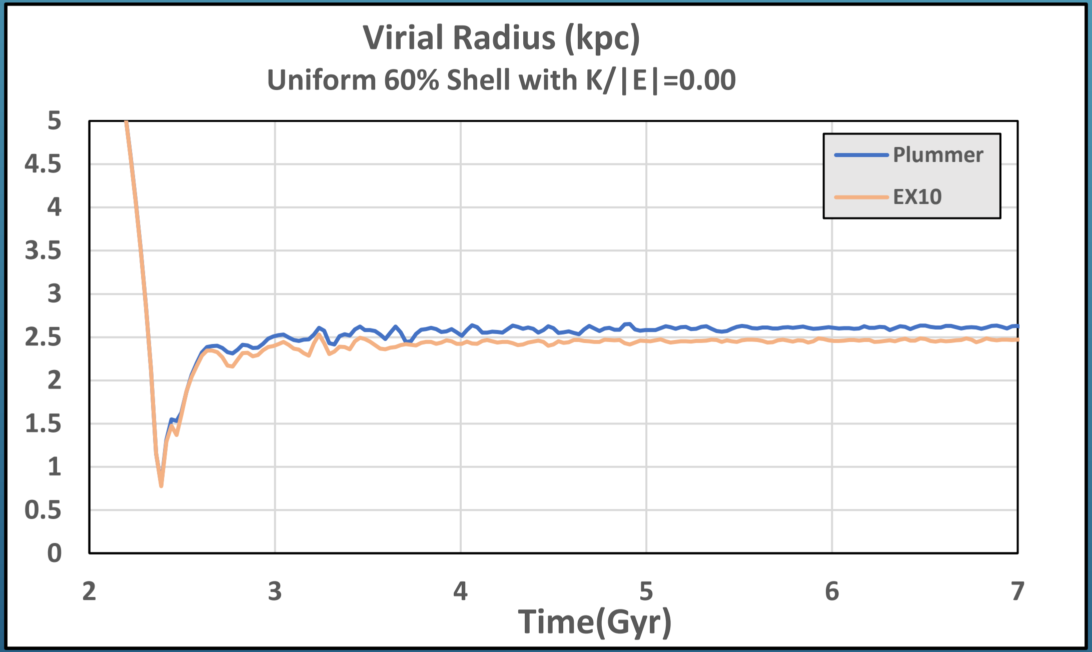
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are over 2000 times the number of force softened pairs, Plummer over EX10.

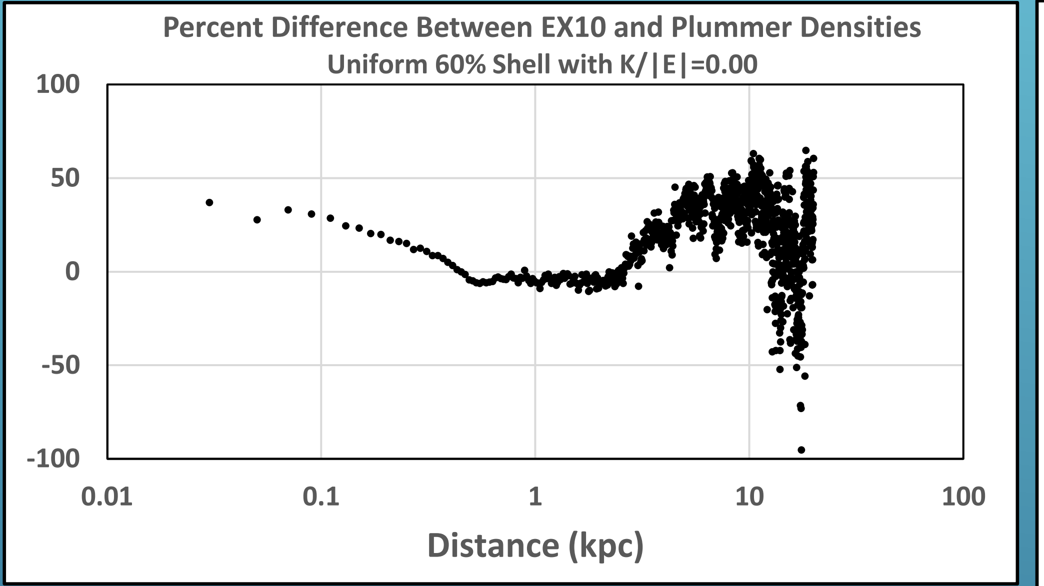
STUDY RESULTS:
COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.00

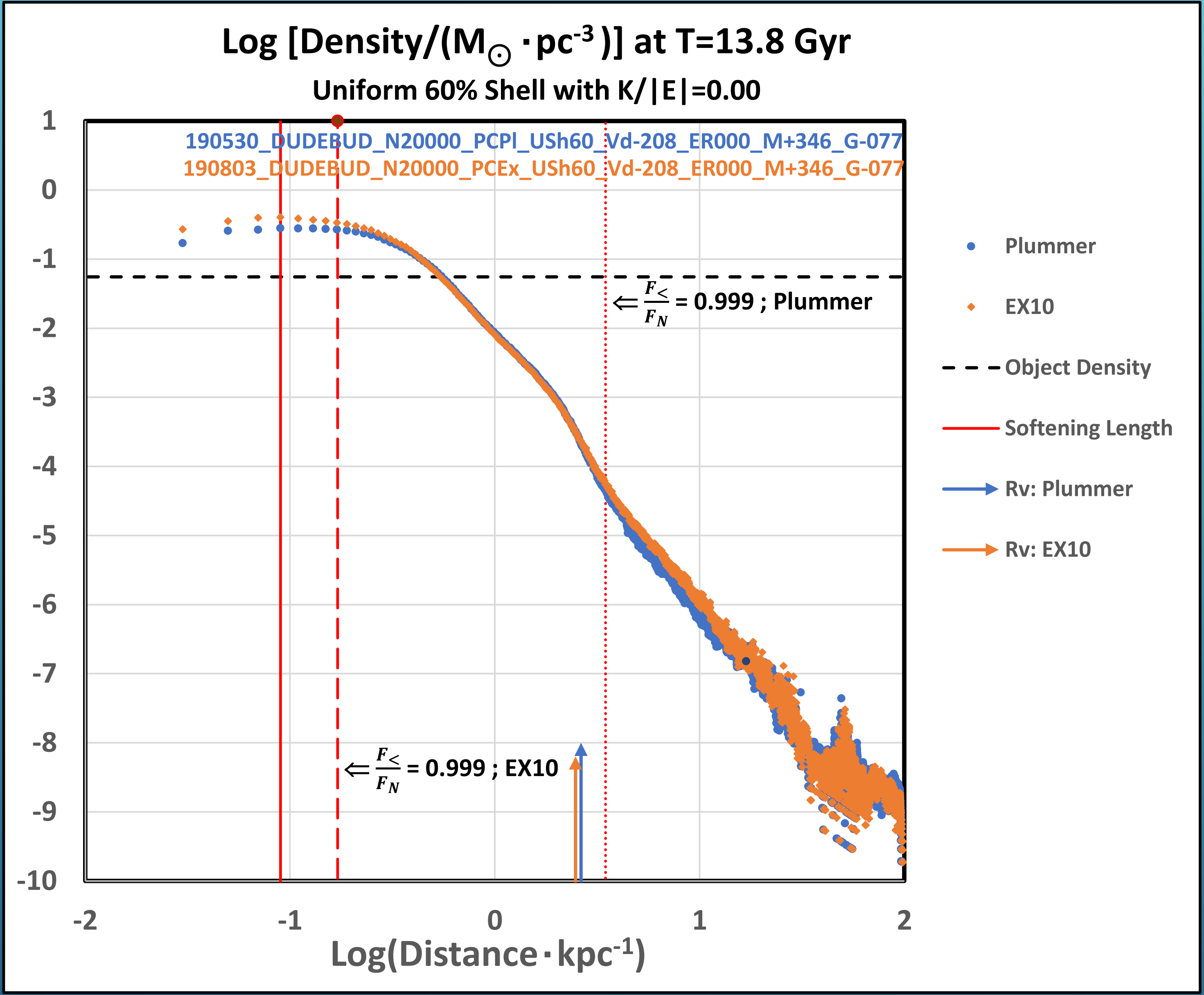
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 7 Gyr, well after each system has achieved a virial condition.
- Note that the virial radius tracks between the Plummer and EX10 simulations out to about 2.7 Gyr.
- Note that the final virial radius for each case was expected to be 8.48 kpc. In this case, difference in the expected and the actual final virial radius is again because of the large numbers of objects ejected from the system.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	13,665	14,129
Ave. number of force-softened pairs	$7.19x10^7$	$9.59x10^5$
Ave. number of particles lost	6335	5870
Final average virial radius	2.46 kpc	2.48 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.00





- The density profiles for the two force softening methods track reasonably well. However, the interior region shows the only significant differences in these methods, where the EX10 densities are larger inside distances of 0.5 kpc.
- The Plummer simulation interior flattens slightly sooner than the EX10 simulation.

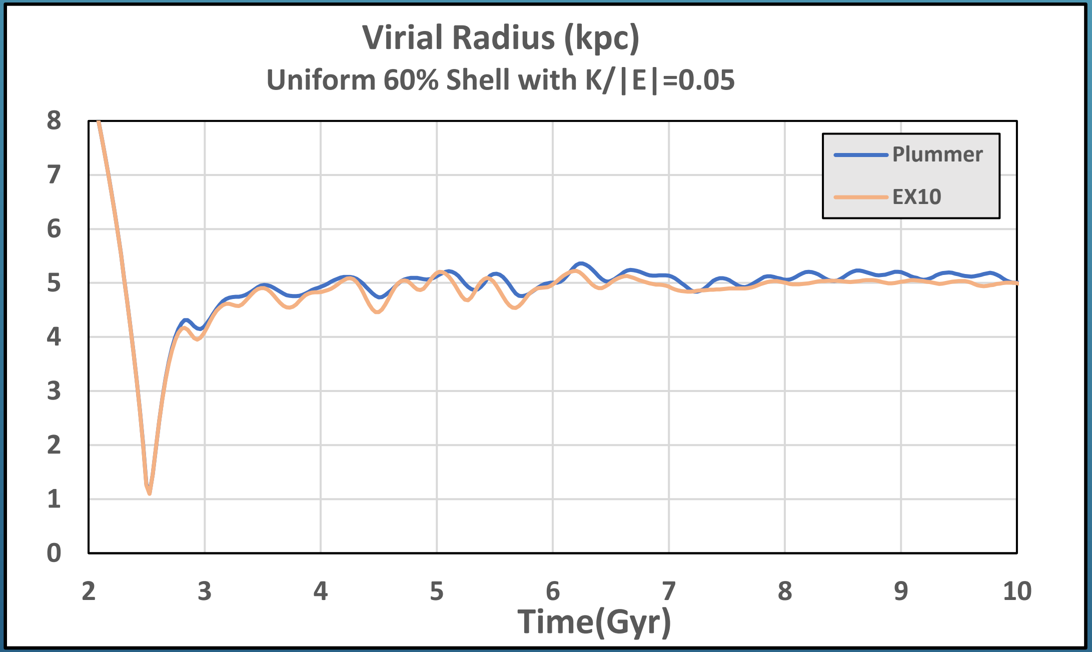
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are about 75 times the number of force softened pairs, Plummer over EX10.

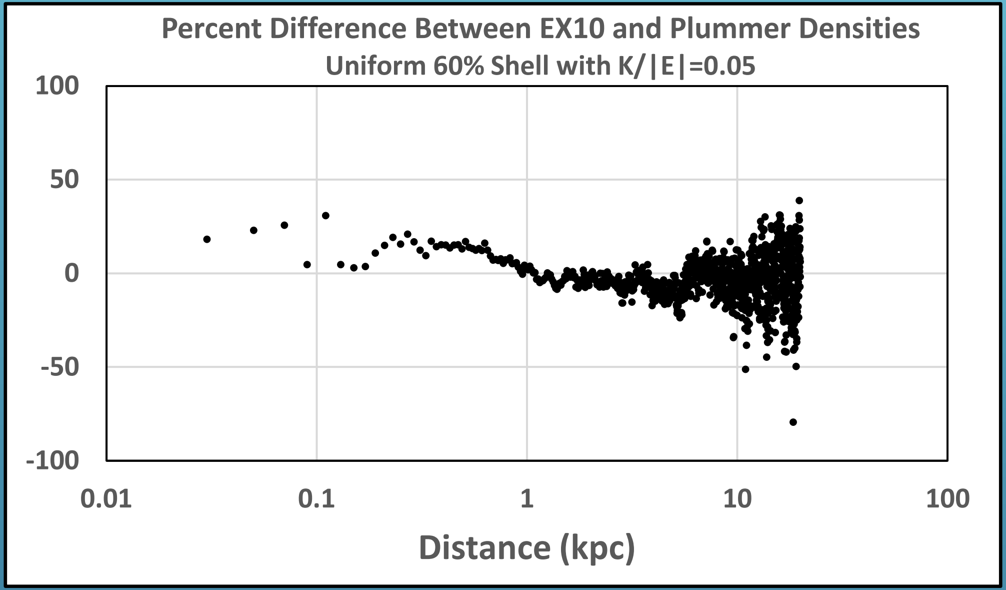
STUDY RESULTS: COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.05

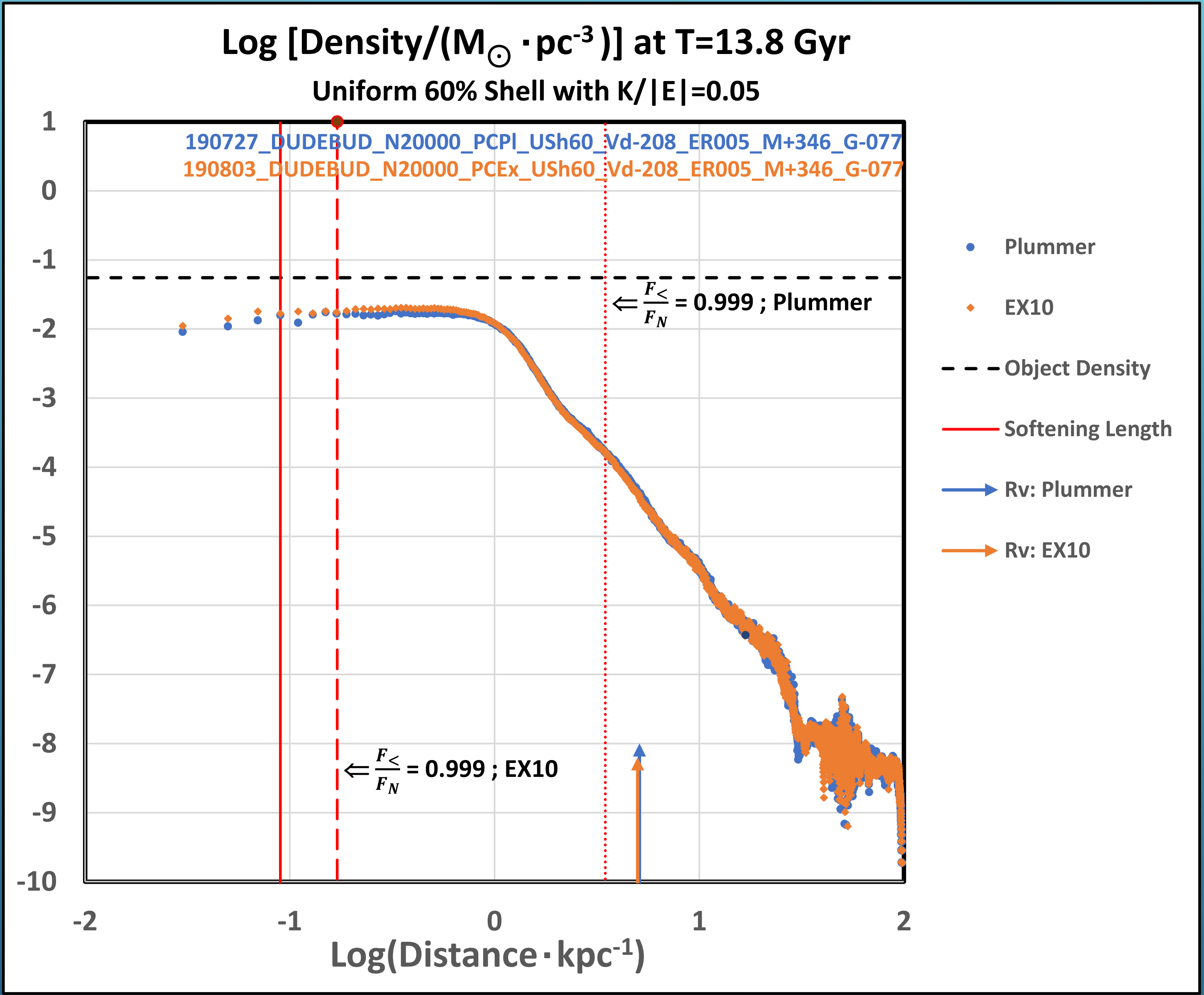
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 10 Gyr. The EX10 simulation reaches a virial state after 7.5 Gyr, whereas the Plummer example takes significantly longer.
- Note that the virial radius tracks between the Plummer and EX10 simulations out to about 2.7 Gyr.
- Note that the final virial radius for each case was expected to be 8.88 kpc. In this case, difference in the expected and the actual final virial radius is again because of the large numbers of objects ejected from the system.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	15,389	15,255
Ave. number of force-softened pairs	4.66×10^{7}	9.25×10^{4}
Ave. number of particles lost	4611	4745
Final average virial radius	5.12 kpc	5.01 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.05





- The density profiles for the two force softening methods track reasonably well. However, the interior region shows the only significant differences in these methods, where the EX10 densities are larger inside distances of 1.0 kpc.
- The Plummer simulation interior flattens perceptively sooner than the EX10 simulation.

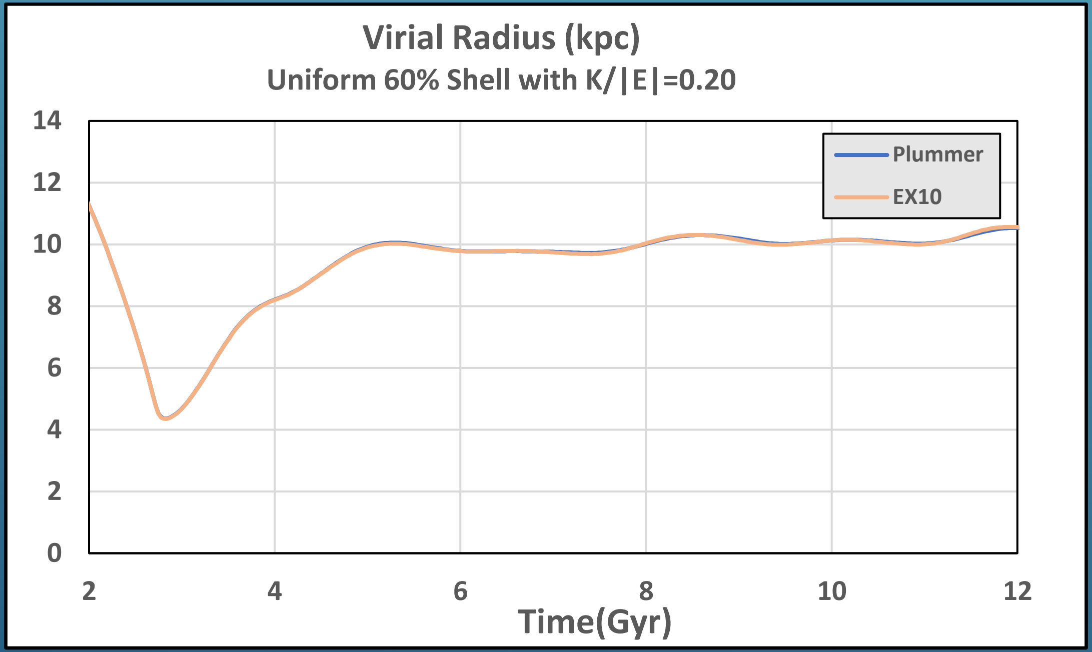
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are over 500 times the number of force softened pairs, Plummer over EX10.

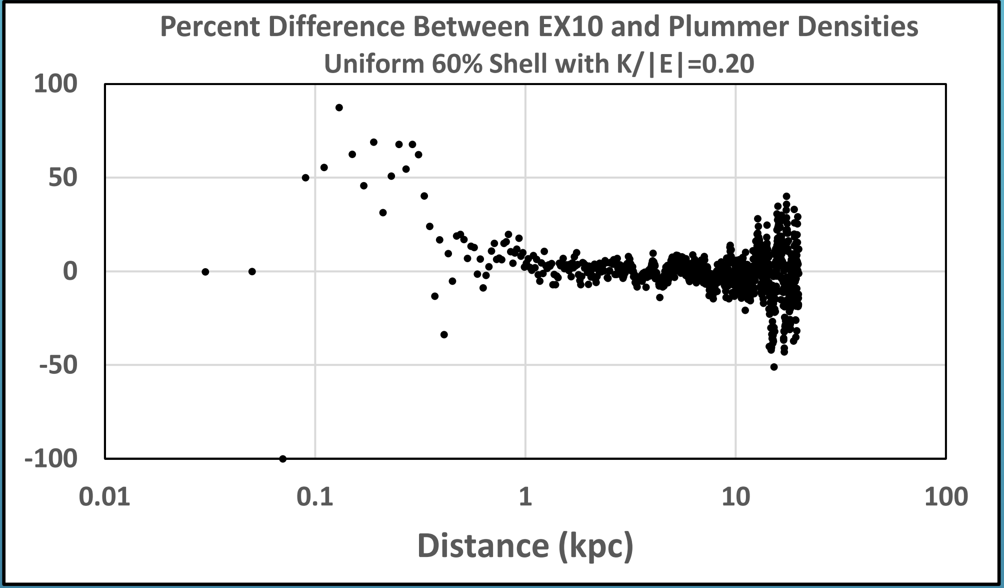
STUDY RESULTS:
COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.20

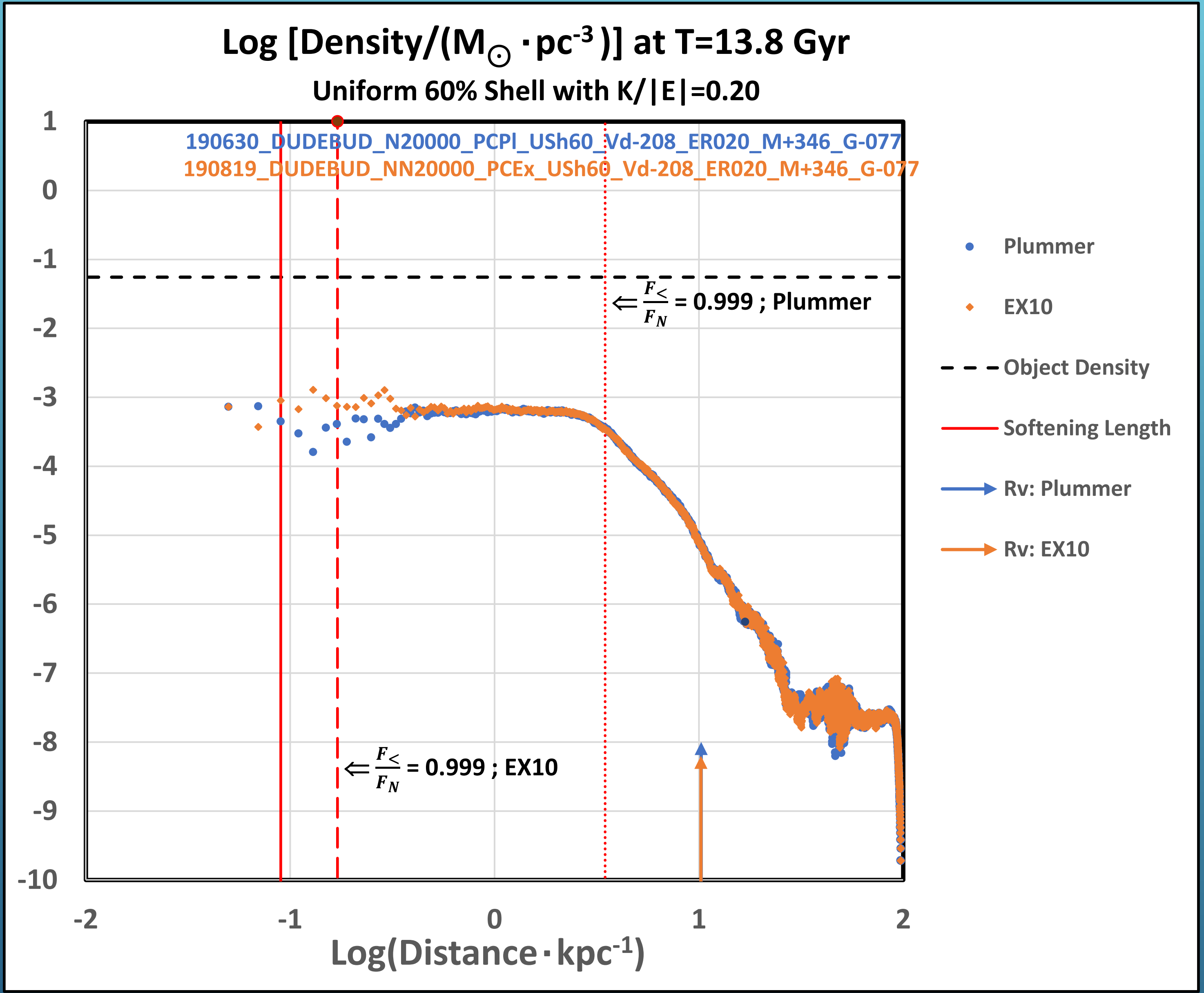
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 12 Gyr. The virial radius of each simulation is still has significant oscillations out beyond 12 Gyr.
- Note that the virial radius tracks between the Plummer and EX10 almost perfectly out to 12 Gyr.
- Note that the final virial radius for each case was expected to be 10.2 kpc, which is in line with the final state virial radius.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	19,371	19,348
Ave. number of force-softened pairs	1.07×10^{7}	3.85×10^{3}
Ave. number of particles lost	630	652
Final average virial radius	10.3 kpc	10.2 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 60% SHELL WITH K/|E|=0.20





- The density profiles for the two force softening methods track reasonably well. However, the interior region shows the only significant differences in these methods, where the EX10 densities are larger inside distances of 1 kpc.
- The Plummer simulation interior flattens perceptively sooner than the EX10 simulation.

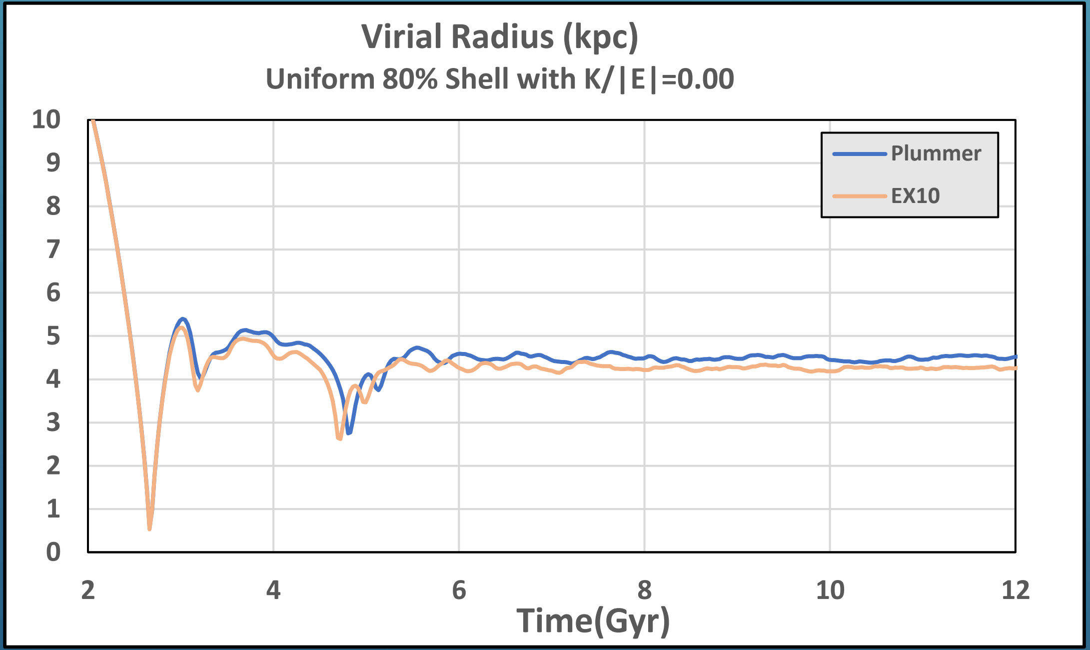
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are about 2800 times the number of force softened pairs, Plummer over EX10.

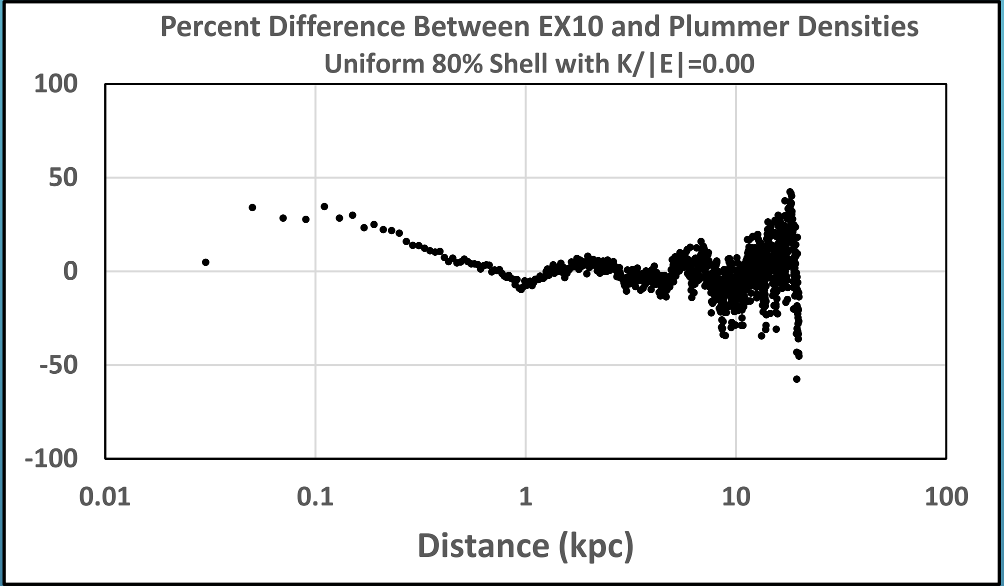
STUDY RESULTS: COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.00

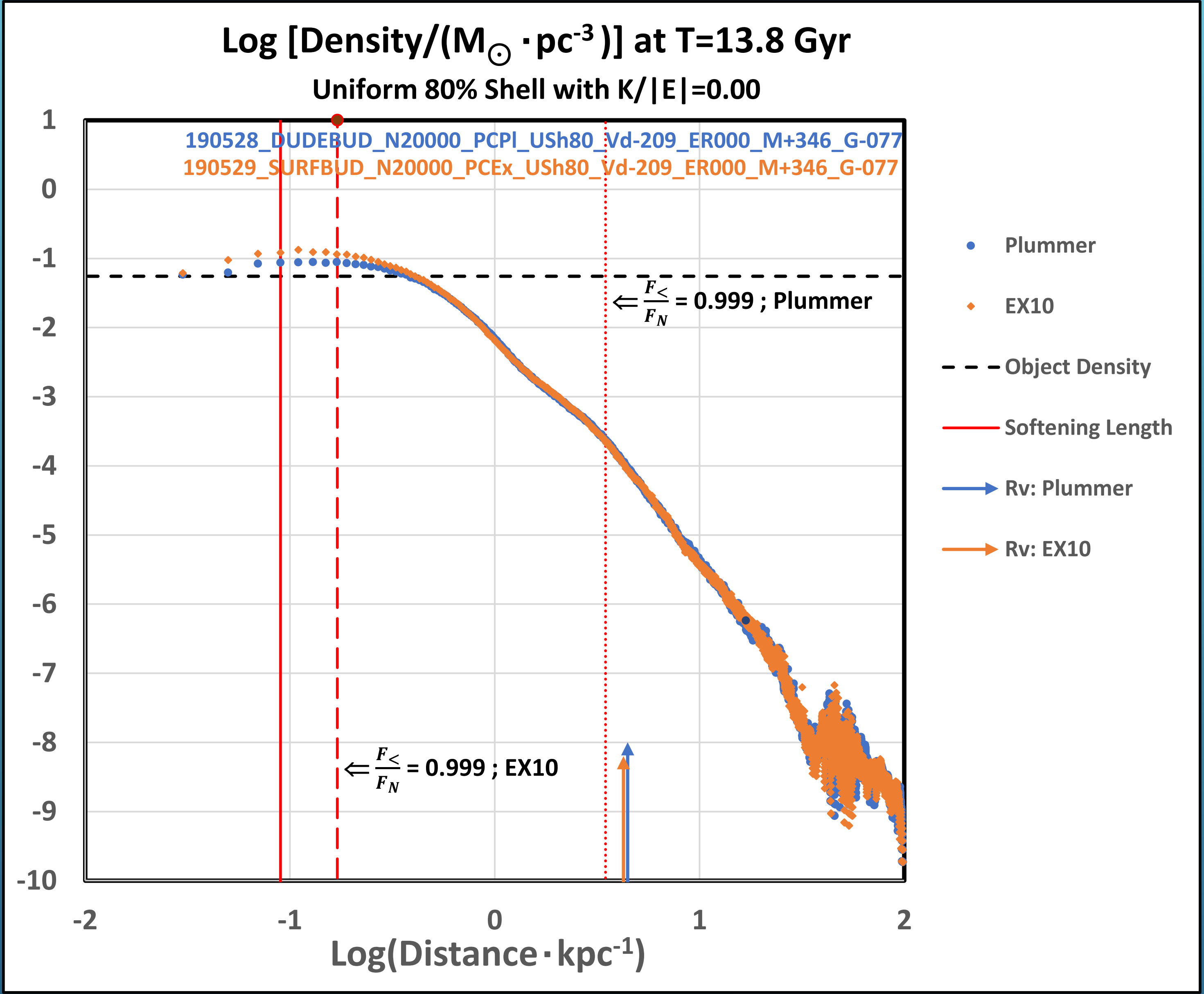
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 12 Gyr. The virial radius of each simulation is still has significant oscillations out beyond 12 Gyr.
- Note that the virial radius tracks between the Plummer and EX10 almost perfectly out to 2.9 Gyr. After that time, the structure looks similar but with a time shift.
- Note that the final virial radius for each case was expected to be 8.9 kpc.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	16,619	16,703
Ave. number of force-softened pairs	4.73×10^7	2.08×10^5
Ave. number of particles lost	3381	3297
Final average virial radius	4.47 kpc	4.26 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.00





- The density profiles for the two force softening methods track reasonably well. However, the interior region shows the only significant differences in these methods, where the EX10 densities are larger inside distances of 1 kpc.
- The Plummer simulation interior flattens perceptively sooner than the EX10 simulation.

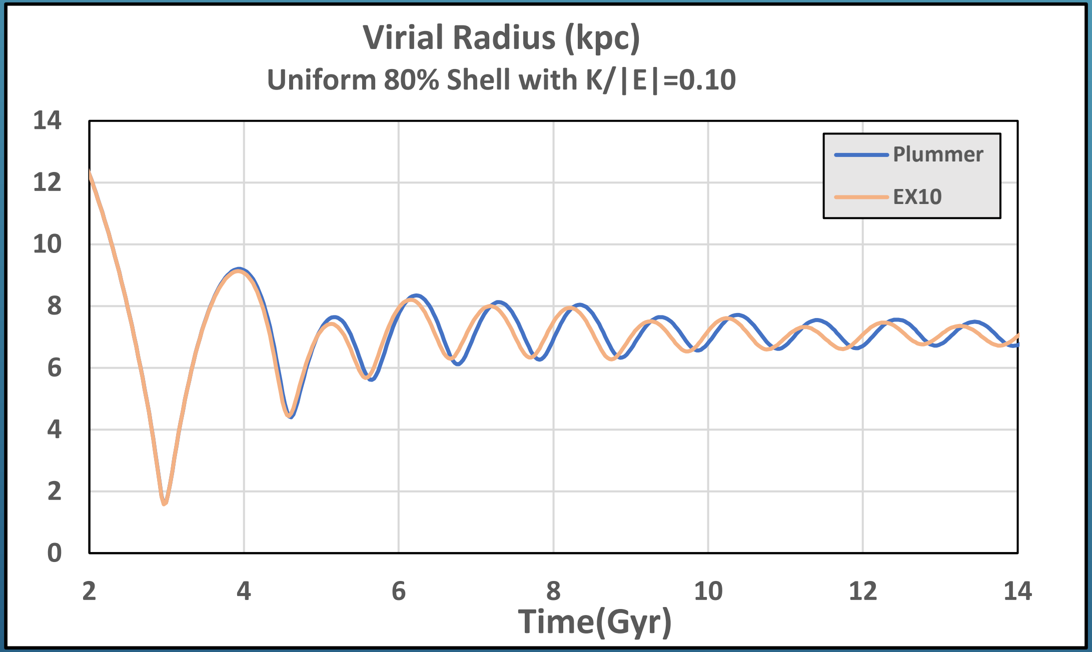
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are almost 230 times the number of force softened pairs, Plummer over EX10.

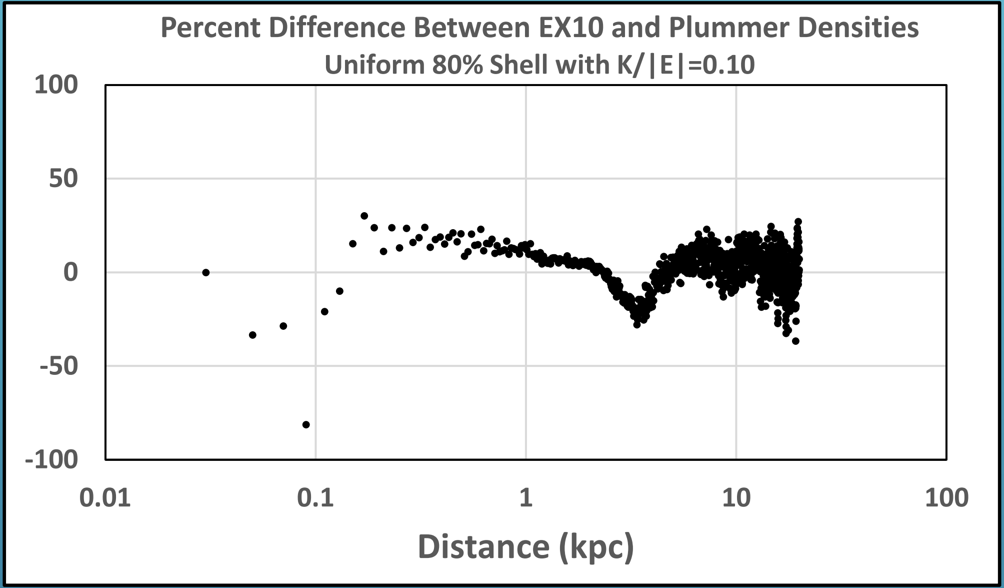
STUDY RESULTS:
COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.10

- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.7 Gyr time.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 14 Gyr. The virial radius of each simulation is still has strong oscillations out to 14 Gyr.
- Note that the virial radius tracks between the Plummer and EX10 almost perfectly out to nearly 4 Gyr. After that time, the oscillatory structure looks similar but with an increasing phase shift.
- Note that the final virial radius for each case was expected to be 9.8 kpc.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	0.5 Gyr	0.5 Gyr
Ave. number particles in histogram	16,961	16,951
Ave. number of force-softened pairs	3.45×10^7	2.16×10^4
Ave. number of particles lost	3039	3048
Final average virial radius	7.01 kpc	6.85 kpc



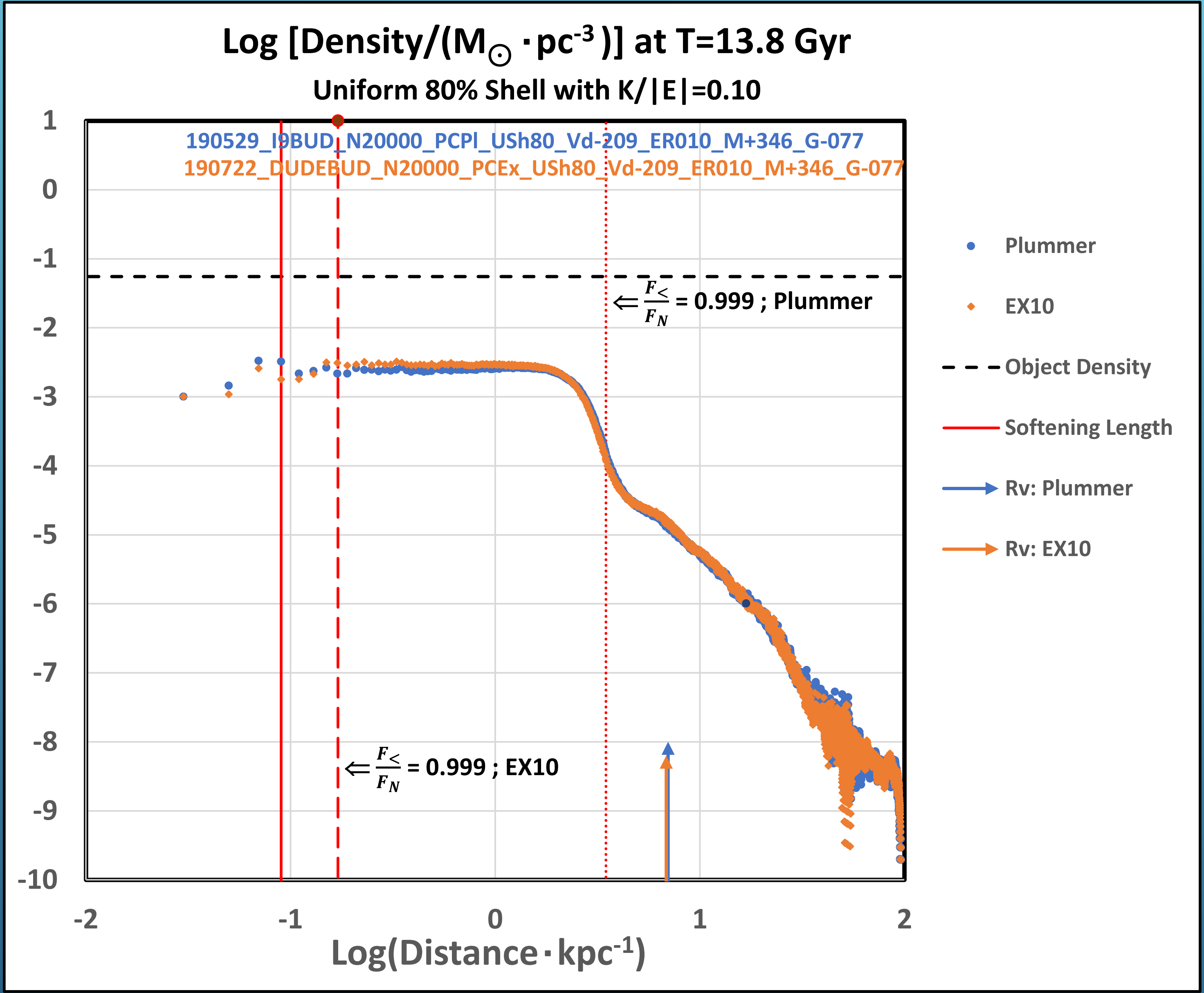
STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.10



- The density profiles for the two force softening methods track reasonably well. However, the interior region shows the only significant differences in these methods, where the EX10 densities are larger inside distances of 2 kpc. The dip between 2.4 and 5 kpc is the only place where the Plummer density profile is larger than the EX10 profile.

Notes:

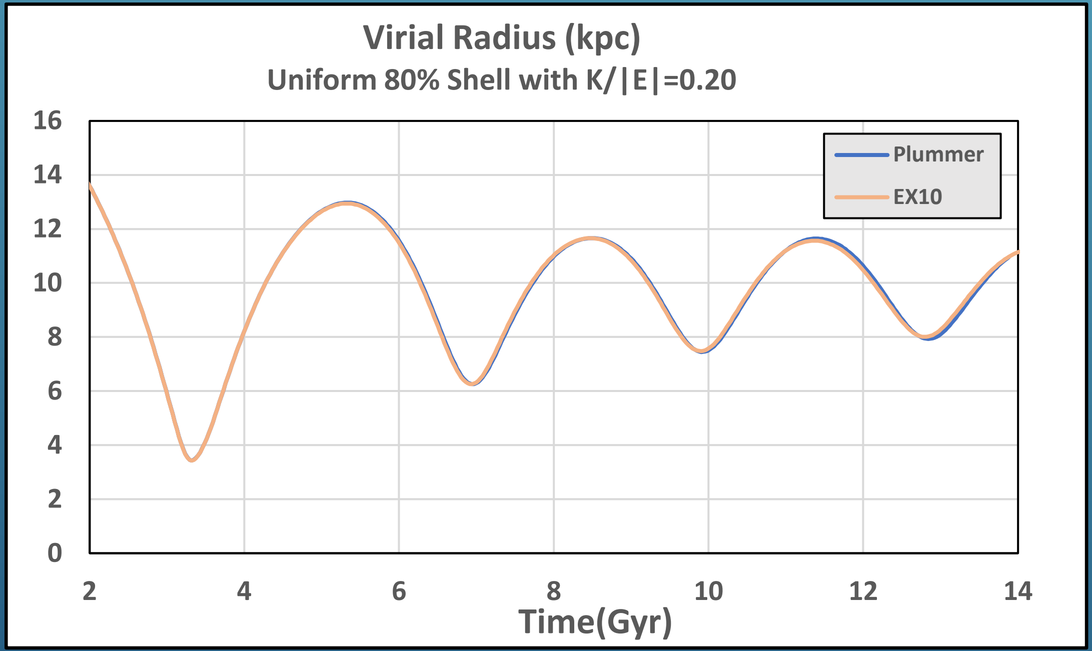
- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are about 1600 times the number of force softened pairs, Plummer over EX10.



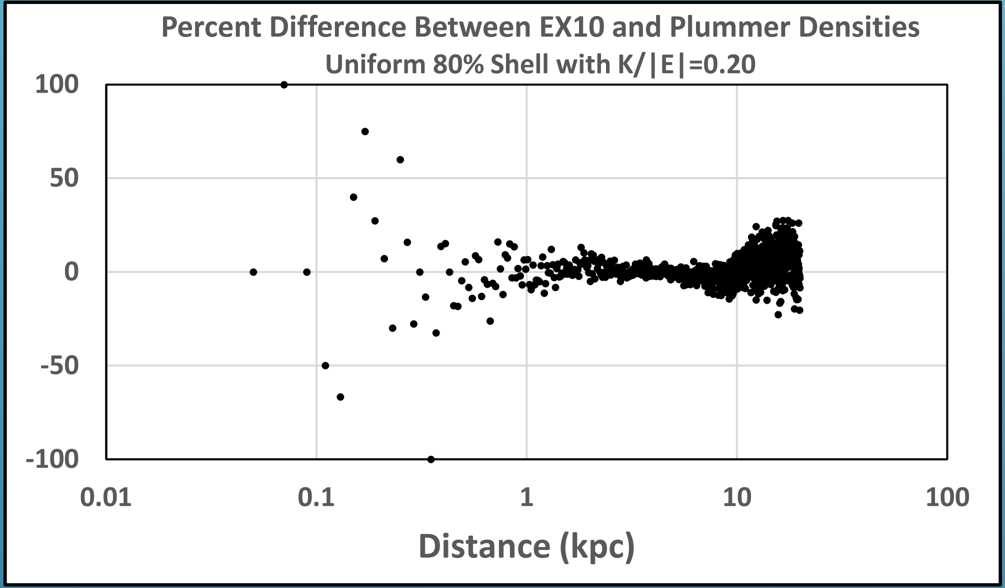
STUDY RESULTS:
COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.10

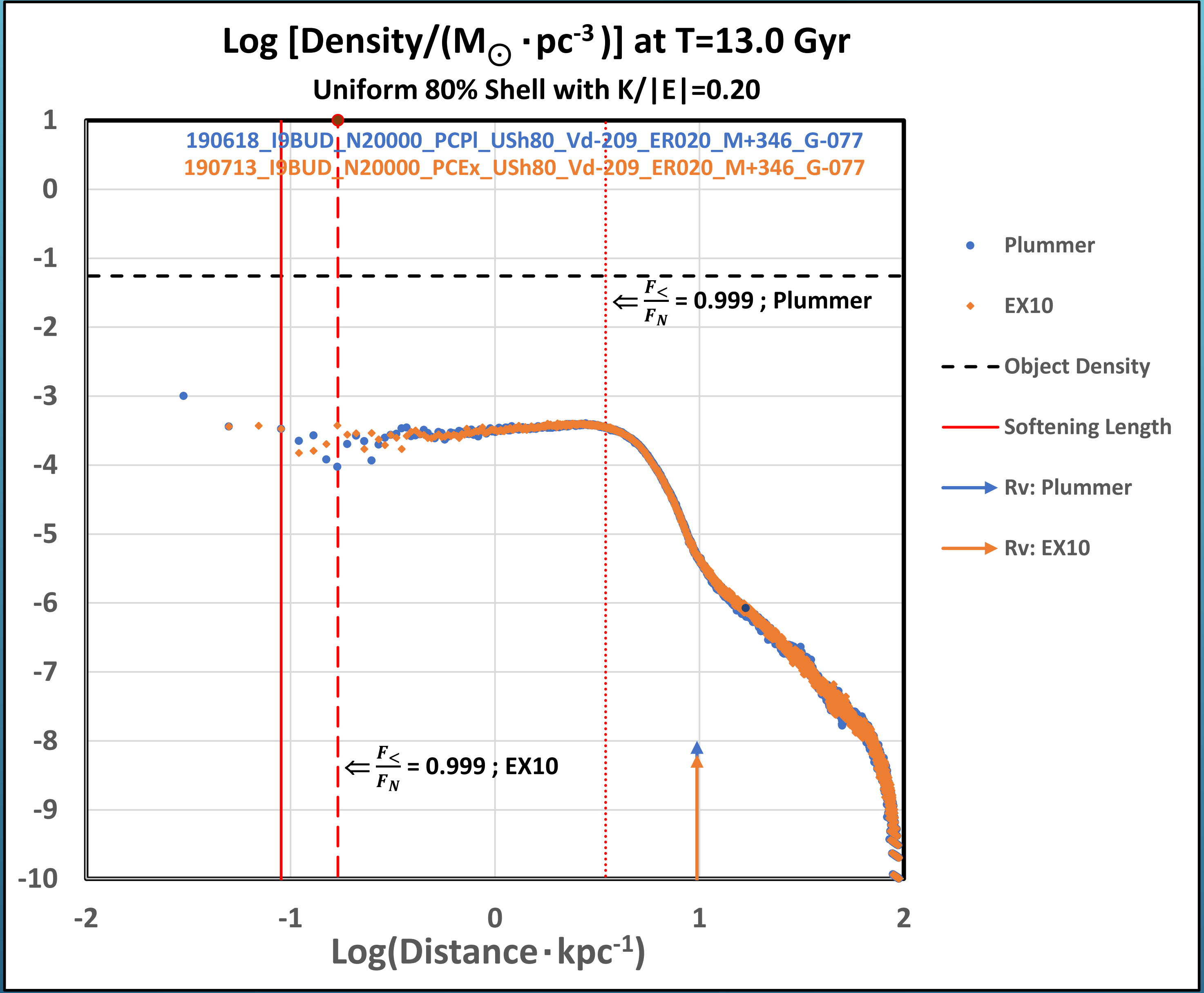
- The table to the right, highlights some of the simulation results for the two simulations, recorded at the 13.0 Gyr time, averaged over a 2.5 Gyr period.
- The plot below shows the virial radius evolution. The scale was reduced to highlight behaviors after the initial collapse at around 2 Gyr out to 14 Gyr. The virial radius of each simulation has very strong oscillations out to 14 Gyr.
- Note that the virial radius tracks between the Plummer and EX10 almost perfectly out to nearly 14 Gyr.
- Note that the final virial radius for each case was expected to be 10.8 kpc.

Simulation Force Softening	Plummer	EX10
Number of event files in Histograms	93	93
Histogram time averaging window	2.5 Gyr	2.5 Gyr
Ave. number particles in histogram	18,535	18,523
Ave. number of force-softened pairs	1.11×10^7	3.70×10^3
Ave. number of particles lost	1466	1476
Final average virial radius	9.7 ± 1.1 kpc	9.7 ± 1.1 kpc



STUDY RESULTS: DENSITY PROFILE COMPARISONS: COLLAPSING UNIFORM 80% SHELL WITH K/|E|=0.10





- There are no real significant differences in these density profiles.

Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- The text at the top are the color coded run strings for each simulation.
- Highlighted in the density profiles are the Newtonian force convergent distances for the Plummer and EX10 methods. All particles inside these limits are experiencing force softening. Note that the previous slide indicates that there are about 3000 times the number of force softened pairs, Plummer over EX10.

AUXILIARY SLIDES

- Simulation features
- Discussion on the virial radius
- Discussion of collision control and force softening
- All Riod details, history and features can found in the users manual at my website: https://riodsim.weebly.com/

THE RIOD SIMULATION FEATURES

- Windows 64bit executable
- Multi-threaded code
- “Unlimited” numbers of particles possible (best if limited to under 100K). The largest simulations I have run are with 50,000 particles.
- Many methods of creating initial conditions, (Density profiles include uniform, shells, flattened disks, Gaussian, EXn, NFW, Jaffe and more.
- Many collision options including elastic, inelastic, Plummer force softening, EX10 force softening and other more exotic types.
- Rich, configurable data logging
- Visualization and analysis tools
- Many other features. See the manual!