RIOD SIMULATION RESULTS
DARK MATTER HALO COLLAPSE SCENARIOS:

Part 2: Uniform Shell Collapse with varying Kinetic Energies

Study Began May 30, 2019

https://riodsim.weebly.com/

DARK MATTER HALO COLLAPSE STUDY MOTIVATIONS

- The results presented here are part of a broad, continuing study, to examine gravitational dark-matter-like collapse scenarios. The idea was to systematically study many scenarios, while attempting to keep only modest changes between simulation runs.
- There are many motivations for beginning this study and I will address some of those below:
  - These collapse scenarios are something that my simulation was easily adapted to do and provides an interesting pastime to contemplate physics topics.
  - It was within the last decade that I discovered that the simulation could adapted and used for "real" cosmological purposes.
  - Curiosity about dark matter as a particle and how might these suspected small objects evolve from the early universe to create galactic halos. Many galactic properties arise from the need for dark matter, constant stellar circular velocities for example.
  - A healthy skepticism about dark matter as a WIMP, a small particle who is its own antiparticle, intrigued me has led to some interesting test scenarios that perhaps will be reported in future studies.
  - Finally, I was curious if I could reproduce halo shapes seen in other, much larger studies. Dark matter halos created in simulations are often compared to density profiles like the NFW, Einasto, and Jaffe profiles.
  - Finally, these initial configuration were kept intentionally simple to provide a learning baseline.

DARK MATTER COLLAPSE STUDY:
PART 1: PARTICLES DISTRIBUTED UNIFORMLY IN A 60% SHELL

- The results reported here begins with the premise: How do particles uniformly distributed within a 60% shell collapse with different initial kinetic energy conditions and what does the final density profile look like after a time corresponding to the age of the universe (13.77 GY)? What I am calling a 60% shell is a spherical shell where the inner radius is 60% of the outer radius.
- As noted in previous results, from a cosmological perspective, this scenario is not particularly relevant. Cosmic Microwave Background data shows that the early universe is a soup of Gaussian density fluctuations. This simulation scenario can best be thought of as an isolated over-density moving in comoving coordinates. In addition, the initial virial density for this study is less than 200 times the current critical density, which implies (I think) that the initial densities are a bit too low, cosmologically speaking.
- The paper from Diemand et.al. (2006) inspired the particle mass and force softening length for this study.
  - Keep number of particles, what I call standard objects (SO) the same (20,000). SO mass is 4.2×10^{34} kg or 21,000 solar masses.
  - Keep the initial sphere radius distance the same at 19.0 kpc.
  - Iterative time slice is 4.33×10^{11} seconds and is the same for all simulations.
  - Use Plummer force softening. Softening length for all simulations is 0.09 kpc. Note, at the time this study was conducted, Plummer and the EX10 (see additional slide at the end) method were being evaluated and compared.

KINECT ENERGY CONDITIONS

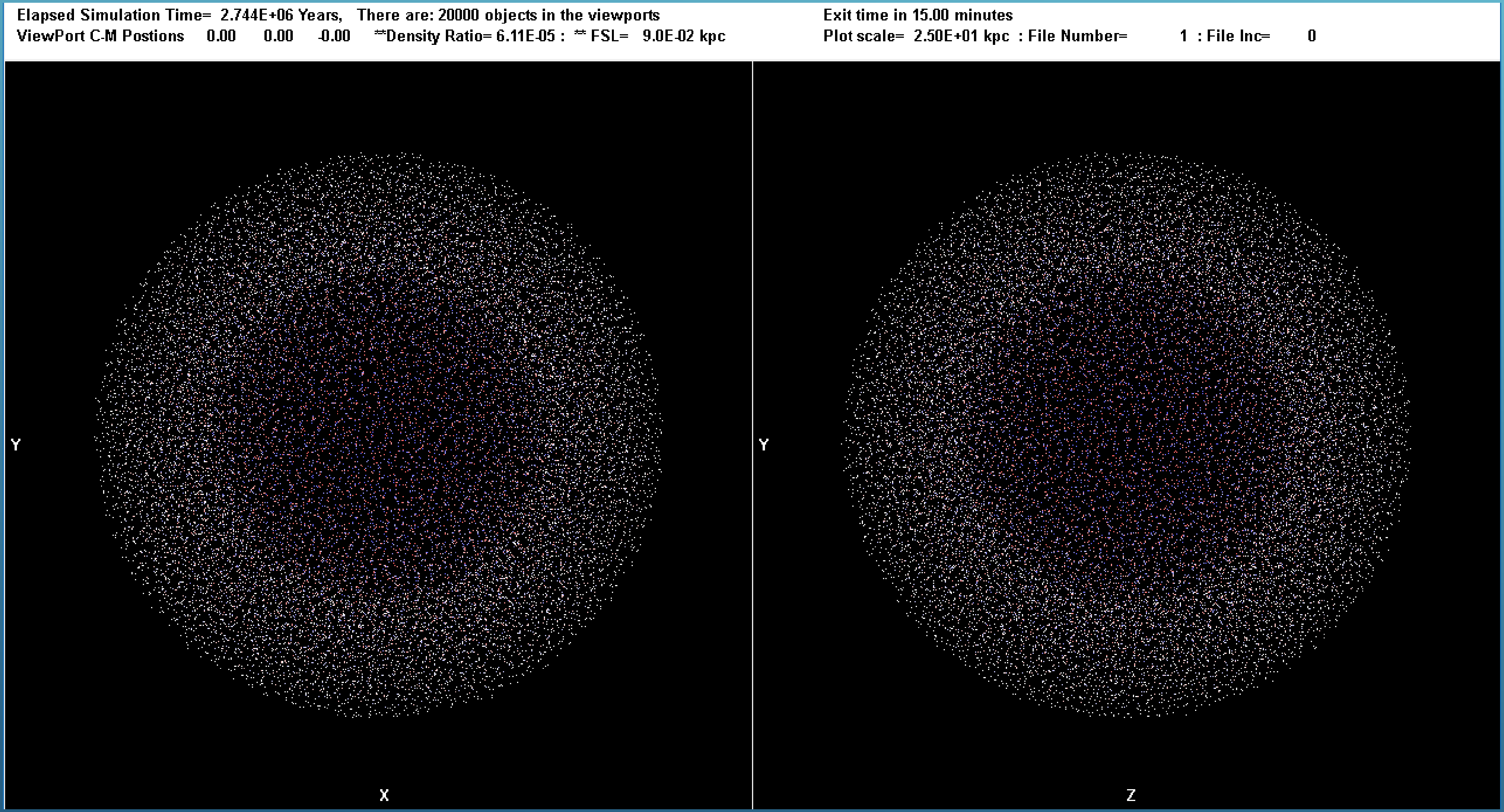
- For this study, each of the four simulations were given a different kinetic energy to total energy ratio, K/|E|. Here K is the total kinetic energy and |E| is the magnitude of the total energy. Note too that for bound systems the total energy in a negative quantity.
- We know that for systems of particles in virial equilibrium, the energy ratio, ER=K/|E|=1.
- Four energy ratios were chosen for this study, ER=0.0, 0.05, 0.10, and 0.20. These four simulations will be referred to as ER000, ER005, ER010, and ER020, respectively.
- From a consideration of initial cosmological conditions, the early universe would have relative kinetic energies close to zero in a comoving group of this size.

PARTICLE CONFIGURATION:

- Visualizing the initial 60% shell configuration for the ER=0.0 configuration:

Notes:

- The scale is 50 kpc edge-to-edge.
- Particles out of the viewing plane are red for closer and blue for father from the viewing plane. Particles that are white are in the viewing plane.



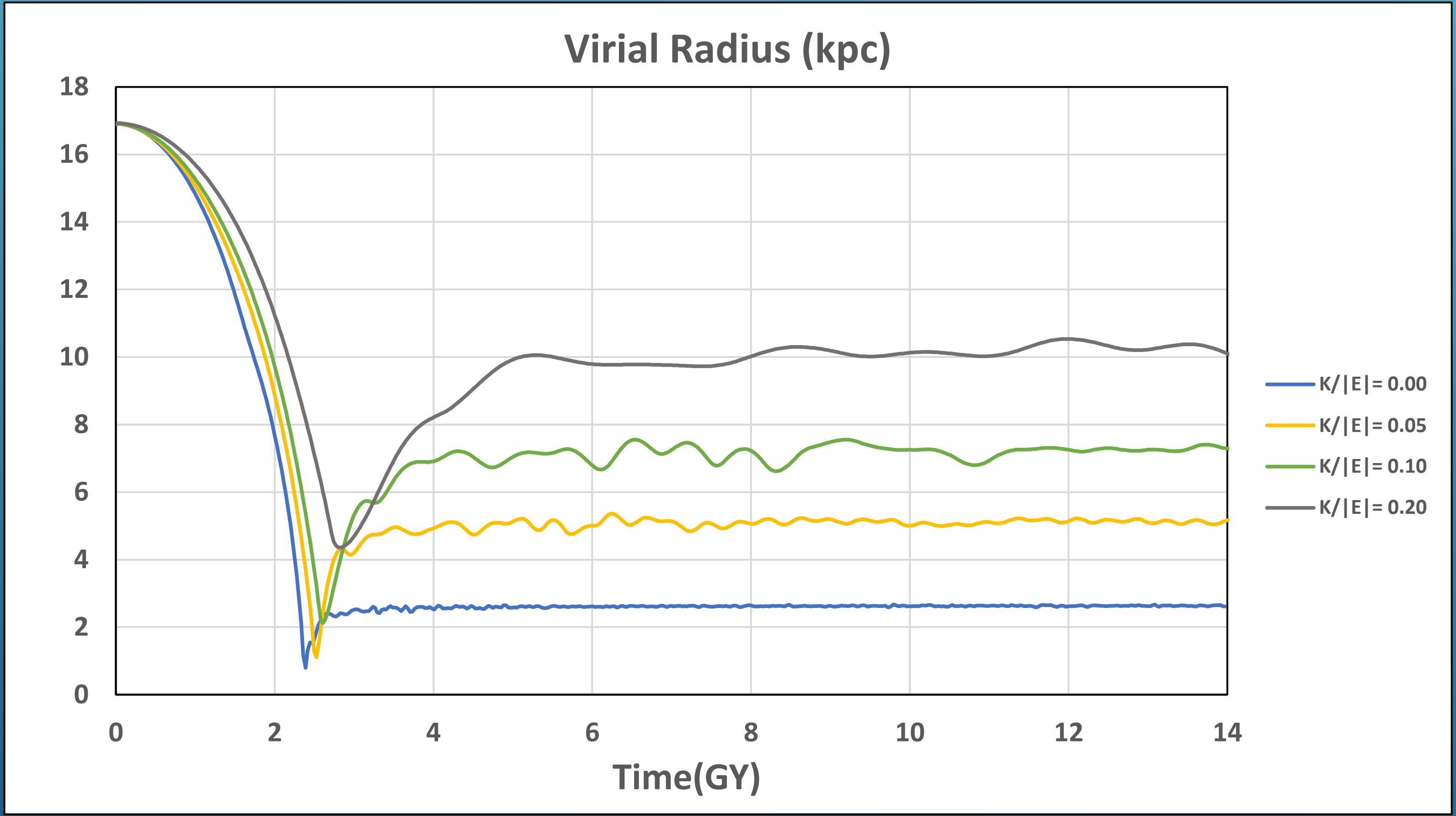
STUDY RESULTS: VIRIAL RADIUS EVOLUTION

- All simulation start with the same virial radius (see end notes for more information on how the virial radius is determined).
- All simulations end with the virial radius significantly affected by the number of particles ejected from the system. Had there been no ejected particles, from the virial theorem, the expected final viral radius in each case is given by the following:

$$r_v = \frac{GM_T^2}{4|E|}$$

Notes:

- Virial state is reached at different times.
- Other than the ER020 simulation, there is a significant number of ejected SO, altering the final virial radius.



STUDY RESULTS: MASS DISTRIBUTION PROFILE AT T=13.77 GY

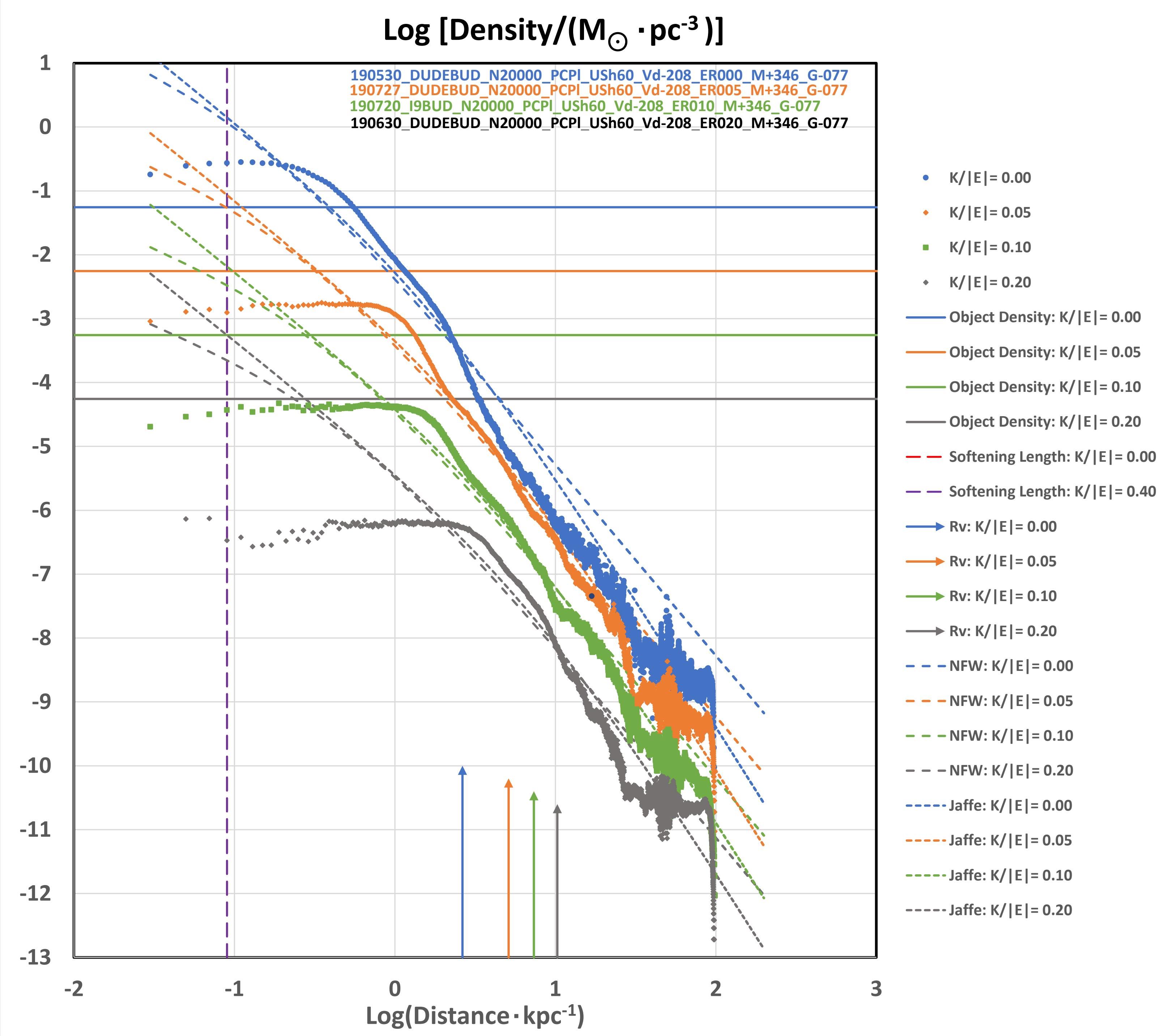
Simulation ID	ER000	ER005	ER010	ER020
Number of event files in Histograms	92	92	92	92
Histogram time averaging window	0.5 GY	0.5 GY	0.5 GY	0.5 GY
Ave. number particles in histogram	13,665	15,389	16,587	19,370
Ave. number of force-softened pairs	$7.19x10^7$	$4.66x10^7$	$2.91x10^7$	$1.07x10^7$
Ave. number of particles lost	6335	4611	3414	630
Final average virial radius	2.64 kpc	5.12 kpc	7.36 kpc	10.3 kpc
NFW200 profile scaling length	0.14 kpc	0.26 kpc	0.38 kpc	0.53 kpc
Jaffe profile scaling length	1.8 kpc	3.5 kpc	5.1 kpc	7.1 kpc

- The ratio of the virial radius and scaling length (r_v/r_s) of an NFW and Jaffe profile has been empirically determined to be 19.6 and 1.45, respectively

STUDY RESULTS: DENSITY PROFILE COMPARISONS

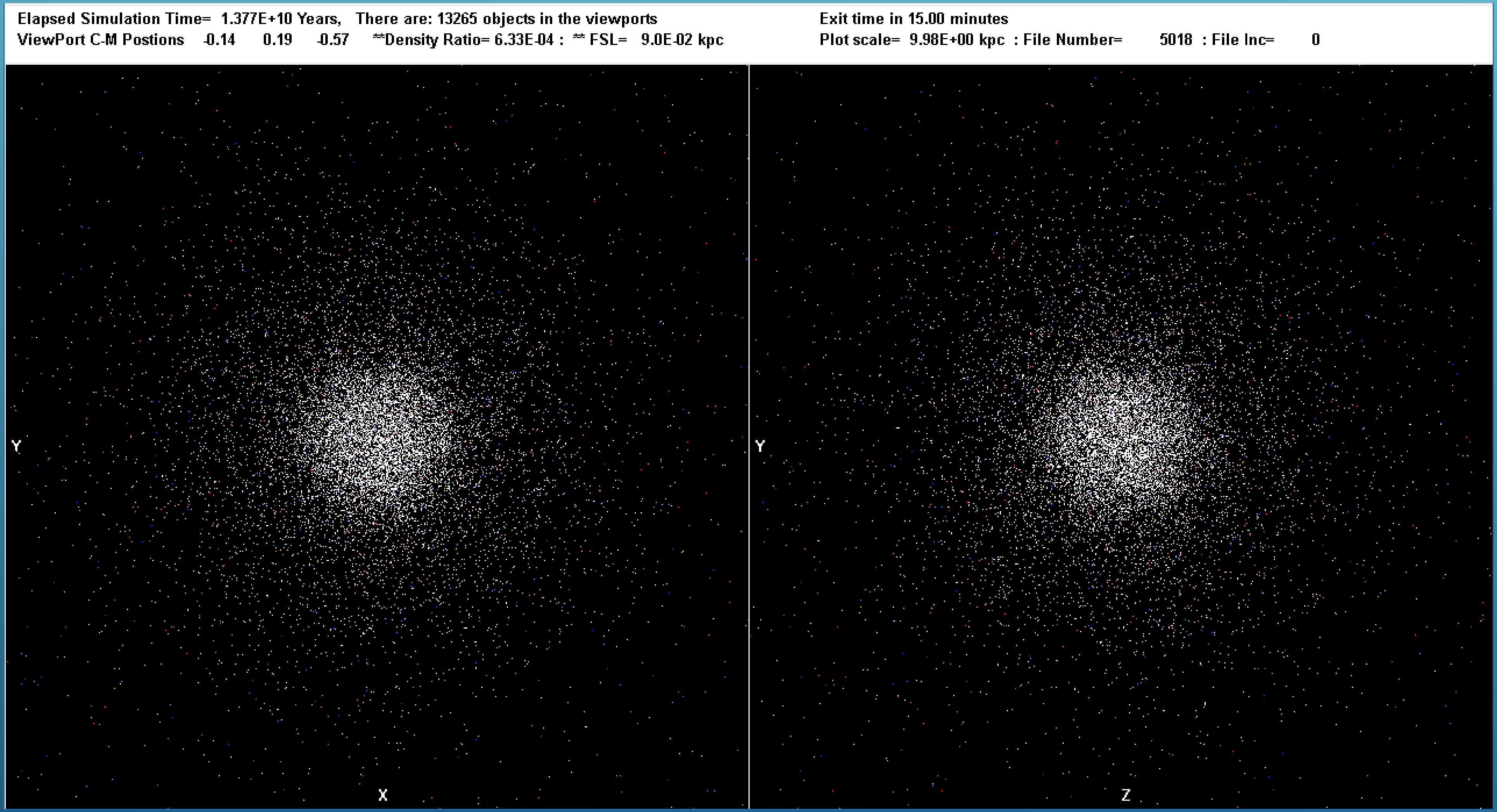
Notes:

- These density profiles are spherically averaged as some of the final profiles are far from spherically symmetric.
- All density profiles are offset by one decade less than the next most upper density profile.
- All particle density lines are offset by one decade less than the next most upper density line
- Particles that are unbound are not included in the histograms that created these density profiles.
- The NFW and Jaffe curves are not fits but based on the final virial radius.
- The softening length is the same for all profiles and is shown for the ER000 case.
- The text at the top are the color coded run strings for each simulation.



EXAMPLE OF A FINAL DENSITY PROFILE

- Here is the ER020 density profile.
- All four final profiles are mostly spherical as seen here.
- All scenarios have a very distinct interior spherical core which has particles distributed uniformly out to a significant range. This example shows the uniform core stretches to about 2 kpc.



STUDY RESULTS (CONT.)

- Observations:
  - Particles starting in a shell of this nature have more potential energy than the uniform sphere case previously reported. More potential energy initially creates more kinetic energy as particles collapse. This accounts for the increased numbers of ejected particles compared to the spherically distributed case.
  - None of these density profiles resemble the NFW or Jaffe profiles; ER005 comes the closest to the NFW shape.
  - The interior density sharply flattens, implying a uniform density in the interior.
  - For the ER000, ER005, and ER010 simulations have significant numbers of SO achieve positive energy and are ejected from the halo, altering the final virial condition.
  - Only the ER000 density profile has maximum densities above or near the individual SO density. There are negative consequences of the high interior density compared to the SO density:
    - First, this becomes a particle-in-a-box packing problem. If there are more particles in a volume than the particle size, then the particles are very tightly packed.
    - Tightly packed particles imply that all particles within that volume are being force softened. What this really means is the physics of the interior is dominated by force softening *not* Newtonian physics which means the halo interior cannot be understood from this simulation scenario.

STUDY RESULTS (CONT.)

- The Plummer force softening method is a long range solution. It requires the interparticle separations to be 39 times the softening length to default to Newtonian physics. As the Plummer method is so long ranged, this increases the number of force softened pairs. For example, in the density profile plots above, all SO inside the distance of 3.5 kpc are actively being force softened with the other SO inside that range.
- One fascinating aspect of using a more compact softening method, like EX10, the density profiles for more localized softening do not differ much from the Plummer method in many cases.   I will revisit this statement in the future.
- Clearly, increased system kinetic energy, increases the time it takes to come to a virial state. For dark matter halos to form and thus create stable galaxies, they must have collapsed with 1-2 GY and come to a virial state quickly (2-4 GY?) after that.
- Systems starting with higher energy ratios exhibit lower interior densities and broader profiles; larger virial radii.
- Another aspect of increasing the collective initial kinetic energy is that fewer particles become unbound and for ER > 0.1, negligible numbers SO are ejected.

STUDY LEARNING POINTS

- Careful choices between particle size/softening length and mass must be considered so that the particle density is well above (an order of magnitude perhaps?) an expected maximum profile density. This isn't always possible since one generally doesn't know what the maximum profile density will be in advance of starting a simulation. This will be a reoccurring theme for future studies.
- Dark matter halos would probably have to collapse quicker than what is seen here and reach a virial state with 4-5 GY. This implies that a higher initial collective density is required. A useful measure for the initial density is a density computed where all the system mass is inside the virial radius, or a virial density. For this study, since all the simulations have the same number of particles of the same mass, they all have the same virial density of $1.76 \times 10^{-21}\ g \cdot m^{-3}$.
- Presumably, greater numbers of SO of lower mass will reduce the percentage of SO that become unbound.

AUXILIARY SLIDES

- Simulation features
- Discussion on the virial radius
- Discussion of collision control and force softening
- All Riod details, history and features can found in the users manual at my website: https://riodsim.weebly.com/

THE RIOD SIMULATION FEATURES

- Windows 64bit executable
- Multi-threaded code
- "Unlimited" numbers of particles possible (best if limited to under 100K). The largest simulations I have run are with 50,000 particles.
- Many methods of creating initial conditions, (Density profiles include uniform, shells, flattened disks, Gaussian, EXn, NFW, Jaffe and more.
- Many collision options including elastic, inelastic, Plummer force softening, EX10 force softening and other more exotic types.
- Rich, configurable data logging
- Visualization and analysis tools
- Many other features. See the manual!

VIRIAL RADIUS DISCUSSION

- The virial radius as defined below becomes an important and useful quantity to help understand these evolving gravitational systems.
- This definition of the virial radius is directly related to the total potential energy of the system and from the virial theorem, we know that once the virial state is reached, the virial radius will be constant (inside a time averaged window).
- For my usage, the following is the definition for the virial radius, r_v:

$$\frac{1}{r_v} = \frac{2}{M_T^2} \sum \frac{m_i m_j}{|\vec{r}_j - \vec{r}_i|} = \frac{2|U|}{M_T^2 G},$$

where U is the total system potential energy. These quantities are easily (but not always quickly) computed from the simulation data.

RIOD SOFTENING METHODOLOGY AND ITERATIVE TIME STEPS

- The Riod simulation has an intimate relationship between the iterative time step and the force softening length.
- A solution for determining an integration time step led organically to defining the particle size and consequently the interparticle force softening length. The following question was asked:
  - "What is the period of orbit for two identical particles orbiting in a circular orbit at their two radii, "*s*".
  - There is a known solution for this query. Using Kepler's third law we know that $T^2 \propto a^3$; where *T* is the orbital period and *a* is the orbital semi-major axis.

  The orbital period is divided into N time slices and that becomes the integration time, $\Delta t = T/N$ then:

$$\Delta t^2 N^2 \propto a^3$$

  Now, for circular orbits of radius *s*, the semi-major axis is also the radius.

  Thus $s=a$.

RIOD SOFTENING METHODOLOGY AND ITERATIVE TIME STEPS (CONT.)

Finally, adding the missing constants, we see that for identical particles, Kepler's law gives us a relationship between the particle size and the integration interval:

$$\Delta t = \frac{\pi}{N}\sqrt{\frac{2s^3}{Gm}}$$

There is much of interest in the above relationship.

- One can specify the particle size to get the time slice or one can specify the time slice and get a particle size. In fact, the Riod simulation gives the operator either option.
- The time slice essentially depends only on the particle density. Thus for unequal masses of the same density, this relationship holds.

After much testing, it was found that a value of N=10,000 gives excellent results.

RIOD SIMULATION: FORCE SOFTENING DETAILS

- Force-softening for this study used the Plummer method. Details of the Plummer method can easily be discovered searching the net.
- Riod has another option for force softening and is based on a highly localized force modification, such that the simulation shuts off all force softening for interparticle separations greater than twice the soften length.
- This interior force ($F_<$) modification is given by (r_l is the softening length):

$$F_< = F_N\left[1 - e^{-(r/r_l)^3}\right],$$

where, F_N is the standard Newtonian force.

Note that $F_</F_N = 0.999$ when r = 1.90 r_l ; thus shutting of force softening at $r = 2.\, r_l$,a reasonable compromise.

This force softening method will be referred to as EX10, as the more general form of the exponential function to a power of 30/n. where n=10 or generically can be written as EXn:

$$\boldsymbol{\rho}(\mathbf{r}) = \rho_0\left[e^{-(r/r_l)^{30/n}}\right]$$

COLLISION CONTROL AND FORCE SOFTENING (CONT.)

Collisions are managed with force modifications, which are visualized below, with the following notes.

- Note that the EX10 (also called smoothed Gauss) force softening converges to the Newtonian force for $r/r_l>1.9$.
- The Plummer softening method does not converge to the Newtonian force until 20 times farther out, or $r/r_l>39$.
- The Lennard-Jones modification (called repulsive core) works well enough for certain powers of n and converges quickly to the Newtonian force.
- The piece-wise continuous elastic collision force modification has issues with its implementation and thus is not recommended for use.